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CONTENTS


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page

RADIO AND LINE TRANSMISSION B, 1964 (Q. 2–10) .. .. .. .. .. .. 1

TELEPHONY B, 1964 .. .. .. .. 4

TELEGRAPHY B, 1964 .. .. .. .. 8

MATHEMATICS C, 1964 .. .. .. .. 12

LINE PLANT PRACTICE B, 1964 .. .. 17

TELECOMMUNICATION PRINCIPLES C, 1964 (Q. 1–6) .. .. .. .. .. .. 21


## CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS, 1964

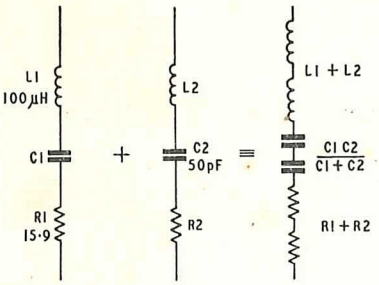
## QUESTIONS AND ANSWERS

**Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.**

### RADIO AND LINE TRANSMISSION B, 1964 (*continued*)

***Q.* 2.** *A series-tuned circuit, resonant at* 1 *Mc/s, has a coil of inductance* 100 *μH and resistance* 15·9 *ohms. Another series-tuned circuit has a* 3*-db bandwidth of* 10 *kc/s and a resonant frequency of* 2 *Mc/s, when tuned by a capacitance of* 50 *pF. The two tuned circuits are then connected in series. Calculate the resonant frequency,* 3*-db bandwidth, resonant impedance and effective Q-factor of the combination assuming that the coil resistances do not change with frequency.*

**A. 2.** The two tuned circuits are shown in the sketch.




If the angular resonant frequency of the first circuit is $\omega_1$ then the value of $C_1$ may be obtained from the following expression:

$$C_1 = \frac{1}{\omega_1^2 L_1} = \frac{10^{12}}{(2\pi \times 10^6)^2 \times 100 \times 10^{-6}} = 253{\cdot}3 \text{ pF.}$$

Similarly, if $\omega_2$ is the angular resonant frequency of the second circuit, then the value of $L_2$ may be obtained from the following expression:

$$L_2 = \frac{1}{\omega_2^2 C_2} = \frac{10^6}{(2\pi \times 2 \times 10^6)^2 \times 50 \times 10^{-12}} = 126{\cdot}7 \ \mu\text{H.}$$

Now, $B$ = Bandwidth $= \dfrac{f_2}{Q_2}$ and, $Q_2 = \dfrac{\omega_2 L_2}{R_2}$, where $f_2 = \omega_2/2\pi$ and $Q_2 = Q$-factor.

Thus $B = \dfrac{f_2}{\omega_2 L_2} R_2$.

$$\therefore R_2 = \frac{B\omega_2 L_2}{f_2} = B \times 2\pi \times L_2$$

$$= 10 \times 10^3 \times 2\pi \times 126{\cdot}7 \times 10^{-6} = 7{\cdot}96 \text{ ohms.}$$

When placed in series the combination is equivalent to a series-tuned circuit of $L_3$, $C_3$ and $R_3$, where

$$L_3 = L_1 + L_2 = 226{\cdot}7 \ \mu\text{H},$$

$$C_3 = \frac{C_1 C_2}{C_1 + C_2} = \frac{253{\cdot}3 \times 50}{253{\cdot}3 + 50} = 41{\cdot}7 \text{ pF},$$

and $R_3 = R_1 + R_2 = 23{\cdot}86$ ohms.

The resonant frequency, $f_3$, is given by the following expression:

$$f_3 = \frac{1}{2\pi\sqrt{L_3 C_3}} = \frac{1}{2\pi\sqrt{226{\cdot}7 \times 10^{-6} \times 41{\cdot}7 \times 10^{-12}}} \text{ c/s}$$

$$= \underline{1{,}636{\cdot}8 \text{ kc/s.}}$$

$$\text{Now } Q_3 = \frac{\omega_3 L_3}{R_3} = \frac{2\pi \times 1{,}636{\cdot}8 \times 10^3 \times 226{\cdot}7 \times 10^{-6}}{23{\cdot}86}$$

$$= \underline{97{\cdot}7}.$$

$$\text{Bandwidth} = \frac{f_3}{Q_3} = \frac{1{,}636{\cdot}8}{97{\cdot}7} = \underline{16{\cdot}75 \text{ kc/s.}}$$

Impedance at resonance $= R_3 = \underline{23{\cdot}86 \text{ ohms.}}$

***Q.* 3.** *In the diagram shown in Fig. 1, the* 500*-kc/s test oscillator is very loosely coupled to the* 300*-μH inductor L. If the circuit is at resonance, calculate the value of* $C_1$.

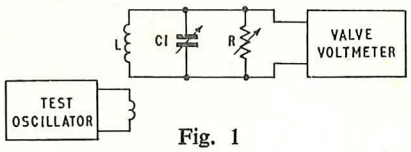



Fig. 1

*When R is adjusted to* 1 *megohm, the valve voltmeter reads* 1 *volt. An additional capacitor,* $C_2$*, is now placed in parallel with the inductance and* $C_1$ *is readjusted to give maximum reading on the voltmeter. R is then readjusted so that the voltmeter again reads* 1 *volt. If the new values of R and* $C_1$ *are* 4 *megohms and* 138 *pF, respectively, calculate for* $C_2$*:*

(*a*) *the capacitance,*

(*b*) *the equivalent r.f. shunt resistance, and*

(*c*) *the power factor.*

**A. 3.** Let $\omega$ be the angular resonant frequency of the circuit.

$$\text{Then, } C_1 = \frac{1}{\omega^2 L} = \frac{10^{12}}{(2\pi \times 0{\cdot}5 \times 10^6)^2 \times 300 \times 10^{-6}} \text{ pF.}$$

$$= \underline{338 \text{ pF.}}$$

Thus $C_1 = C_1 - C_1' = 338 - 138 = \underline{200 \text{ pF}}$,

where $C_1'$ is the new value of $C_1$.

If the shunt resistance of the capacitor under test is $R_x$, as the final resistance value is unchanged from the initial value,

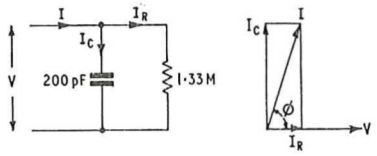
then $\dfrac{R' \times R_x}{R' + R_x} = R$,

where $R' = 4$ megohms = new value of $R$.

$\therefore (R_x + 4) \times 1 = 4 R_x$

$\therefore \underline{R_x = 1\tfrac{1}{3} \text{ megohms.}}$

The sketch shows the equivalent circuit of the capacitor and the corresponding vector diagram.




The power factor of the capacitor is cos $\phi$, and because $I_C >> I_R$, cos $\phi$ = cot $\phi$ with negligible error.

Thus, power factor = $\cot \phi = \dfrac{I_R}{I_C}$,

$$= \frac{V}{R_2} \times \frac{1}{V\omega C_2} = \frac{1}{\omega C_2 R_2},$$

$$= \frac{1}{2\pi \times 0{\cdot}5 \times 10^6 \times 200 \times 10^{-12} \times 1{\cdot}33 \times 10^6}$$

$$= \underline{1{\cdot}19 \times 10^{-3}}.$$

***Q.* 4.** *Draw a block diagram of a communications-type superheterodyne radio receiver and briefly explain the function of each stage.*

*Discuss any inherent disadvantage of the superheterodyne principle.*

***Q.* 5.** *Using the valve data given in the table below, plot a family of anode characteristics, using scales of about 1 inch = 50 volts and 1 inch = 10 mA.*

*State whether the valve is a triode or a pentode.*

*Draw load lines for load resistors of 25,000 ohms, 12,500 ohms and 5,000 ohms with h.t. of 250 volts.*

*Say (a) which of these load resistor values would give least distortion for an input signal of amplitude 1 volt and (b) what value of standing bias would be appropriate. Give reasons for your answers, and calculate the stage gain in these circumstances.*

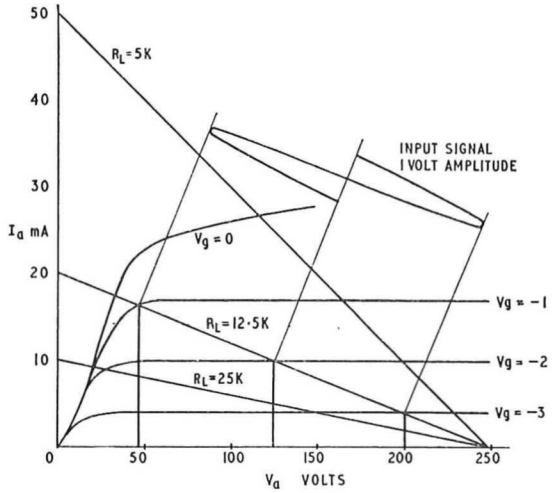
| $V_a$ | $V_g = 0$ $I_a$ | $V_g = -1$ $I_a$ | $V_g = -2$ $I_a$ | $V_g = -3$ $I_a$ |
|---|---|---|---|---|
| 20 | 9 | 9 | 9 | 3·5 |
| 40 | 21 | 15·5 | 9·6 | 4 |
| 60 | 24 | 17 | 10 | 4 |
| 80 | 25 | 17 | 10 | 4 |
| 100 | 26 | 17 | 10 | 4 |
| 150 | 28 | 17 | 10 | 4 |
| 200 | — | 17 | 10 | 4 |
| 250 | — | 17 | 10 | 4 |

**A. 5.** The family of anode characteristics is shown in the sketch. For convenience in printing, the scale is smaller than that asked for in the question.

The curves show that the valve behaves as a constant-current generator for a wide range of anode voltages. This behaviour is typical of a pentode.

Load lines for load resistors of 25,000 ohms, 12,500 ohms and 5,000 ohms with an h.t. of 250 volts are shown in the sketch.

For low distortion the positive and negative excursions of the anode voltage should be as nearly equal as possible. The load resistor should be chosen to give as high a stage gain as possible while still allowing the valve to operate within the linear portion of its characteristic. For these reasons (*a*) the best value of load resistor for an input signal of amplitude 1 volt would be 12,500 ohms and, (*b*) the value of standing bias should be $\underline{-2 \text{ volts}}$. These values fulfil the necessary conditions.




The stage gain is the ratio of the output signal amplitude to that of the input signal. From the graph, the magnitude of the output voltage swing, along the 12,500-ohm load line for an input voltage swing of ± 1 volt, is seen to be 155 volts. The stage gain is therefore $\dfrac{155}{2} = \underline{77{\cdot}5}$.

***Q.* 6.** *Show that the gain of an amplifier with negative feedback may be given by the expression:*

$$\mu_F = \frac{\mu}{1 + \mu\beta}$$

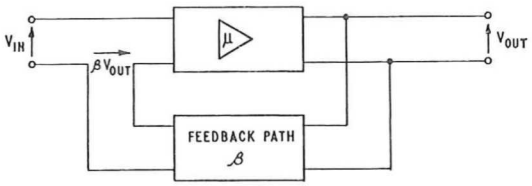
*where* $\mu_F$ = *gain with feedback,*

$\mu$ = *gain without feedback, and*

$\beta$ = *fraction of output voltage fed back to input.*

*An amplifier has a gain without feedback of 54·8 db. what will be the gain when the feedback path, having a* $\beta$ *of 0·0082, is connected? If the gain without feedback rises by 6 db, what will be the new gain with feedback, expressed in decibels?*

**A. 6.** The arrangment of a feedback amplifier is shown in the sketch. The voltage at the output of the feedback path is $\beta_0 V_{UT}$ and is arranged




to be in opposition to $V_{IN}$. The voltage applied across the input of the forward path is therefore $V_{IN} - \beta V_{OUT}$. The output voltage is then $\mu(V_{IN} - \beta V_{OUT})$ so that

$$V_{OUT} = \mu(V_{IN} - \beta V_{OUT}).$$

Rearranging, $V_{OUT}(1 + \mu\beta) = \mu V_{IN}$,

and $\dfrac{V_{OUT}}{V_{IN}} = \mu_F = \underline{\dfrac{\mu}{(1 + \mu\beta).}}$

If the gain without feedback = 54·8 db and $\beta$ = 0·0082 then 20 log $\mu$ = 54·8 and $\mu$ = 550.

$$\text{Now, } \mu_F = \frac{550}{1 + 550 \times 0{\cdot}0082} = \frac{550}{5{\cdot}5} = 100.$$

The gain, expressed in decibels, is given by

$20 \log \mu_F = \underline{40 \text{ db.}}$

If the gain without feedback rises by 6 db then $\mu = 2 \times 550 = 1{,}100$.

$$\text{Now, } \mu_F = \frac{1{,}100}{1 + 1{,}100 \times 0{\cdot}0082} = \frac{1{,}100}{10} = 110.$$

The gain, with feedback, expressed in decibels, is given by

$$20 \log \mu_F = \underline{40{\cdot}84 \text{ db.}}$$

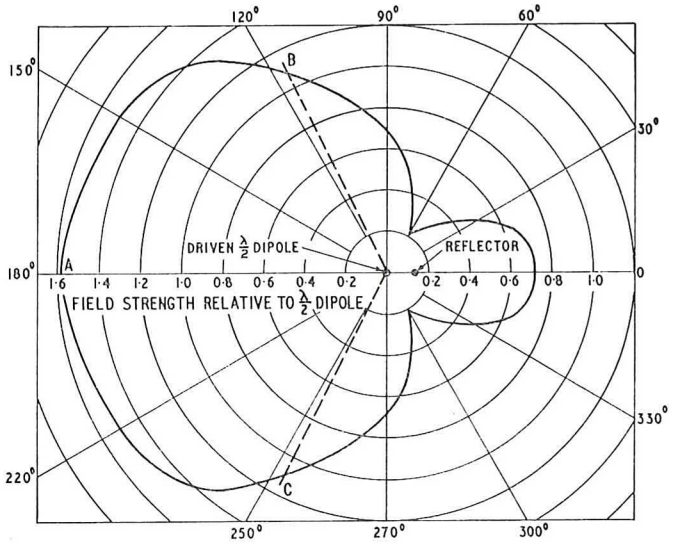
*Q.* 7. *Briefly explain the following terms used in connexion with aerials: "isotropic aerial," "forward gain," "beam width." Sketch, with approximate scales, a radiation diagram in the horizontal plane for a vertical quarter-wave dipole aerial and reflector. Show how this diagram indicates the forward gain and beam width of the array.*

*A resonant dipole aerial is designed to radiate at a particular frequency. Discuss its effectiveness as a radiator at other frequencies.*

A. 7. *Isotropic aerial.* This is a hypothetical aerial which radiates equally in all directions. Although not physically realizable it is a convenient reference against which to compare the gain of other aerials.

*Forward gain.* The gain of a directional aerial may be expressed as the ratio of the field strength produced by the aerial at any point in a given direction to the field strength produced at the same point when the aerial is replaced by a reference aerial fed with the same power. The forward gain is the gain in the direction of maximum radiation.

*Beam width.* The angle included between the direction on each side of the direction of maximum radiation at which the gain is a specified number of decibels below the forward gain is called the beam width. The 3-db, or half-power points, are frequently used, but there is no reason why other points should not be used provided they are specified. In an amplitude polar diagram the relevant points are those at which the amplitude of the field has fallen to 0·707 of its maximum value.



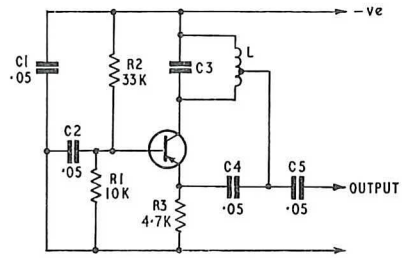

The radiation diagram, shown in the sketch, is drawn using polar co-ordinates. The concentric circles represent various field strengths relative to the the reference aerial; here, a half-wave dipole aerial has been chosen as reference. The radial lines indicate directions of radiation. The beamwidth at the 3-db points is given by the angle BOC and the forward gain by the length OA. Their values are seen to be 158°and 1·58 respectively. It should be noted that the radiation diagram shown and the figures quoted for beam width and gain are only one set from a large variety which may be obtained from such an aerial array, depending on the precise lengths of the elements and their spacing.

The elements of a dipole are normally made a quarter wavelength long at the operating frequency. At twice this frequency they will be a half wavelength long and therefore again resonant. Similarly they will resonate at higher harmonically-related frequencies. Thus the dipole can be used as a radiator (or receiving aerial) at frequencies harmonically related to its design frequency, although impedance matching problems may arise.

A band of frequencies about the resonant frequency will be radiated efficiently and this bandwidth is largly determined by the ratio of the length to diameter of the aerial elements. At frequencies well removed from the resonant frequencies the radiation will be small.

*Q.* 8. *Draw the circuit diagram of a Colpitts or Hartley oscillator using a junction transistor. Briefly describe its operation and give suitable component values if the required frequency is* 1 *Mc/s. List some of the factors which influence the frequency stability.*

A. 8. The circuit diagram of a Hartley-type transistor oscillator is given in the sketch. Bias stabilization is obtained by the potential




divider, $R_1$ $R_2$, across the d.c. supply and the emitter resistor $R_3$. So far as h.f. currents are concerned the base is connected to the top end of the tuned circuit, L $C_3$, because the by-pass capacitors $C_1$ and $C_2$ are of low impedance at the working frequency. Thus, with the emitter connected to the tapping point via capacitor $C_4$ the circuit operates as a Hartley oscillator with the output voltage developed across resistor $R_3$.

The calculation of the exact frequency of oscillation in this type of circuit is complicated because the transistor parameters are involved. However, provided a low value of $L/C_3$ is used, i.e. the tuned circuit is of relatively low impedance, the effect of the transistor is small and the frequency will approximate to the resonant frequency of the tuned circuit. If $C_3$ is 50 pF, then L should have a value of 507 microhenries for a frequency of 1 Mc/s.

The frequency of oscillation is affected by the following factors:

(*a*) Variation of L and $C_3$ with temperature or humidity.

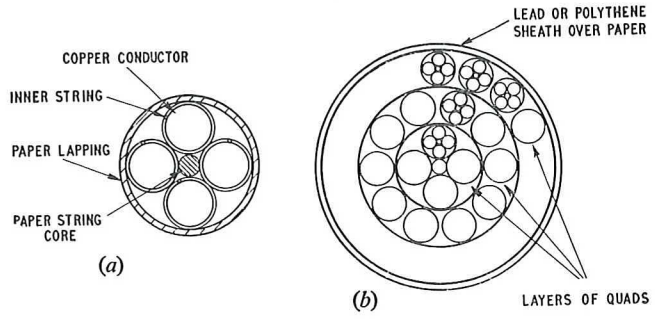
(*b*) Variation of transistor capacitances with operating conditions.

Transistor conditions may be effected by changes of bias, changes of temperature, changes of supply voltage and changes of load impedance.

*Q.* 9. *Sketch in detail the construction of* (*a*) *a multi-pair star-quad telephone cable and* (*b*) *a multi-tube coaxial cable.*

*State the materials used, give typical dimensions and characteristic impedances, and indicate for what types of service these cables are normally used.*

A. 9. (*a*) To produce a quad, individual copper conductors are lapped with paper insulant and four such conductors are formed into a square and twisted together round a paper-string core. Another paper lapping binds the components of the quad together which then has the form shown in sketch (*a*). Numbers of such quads are assembled in layers to form the complete cable, whose cross section is shown in sketch (*b*).




Each layer is held in place by a cotton whipping. A final overall sheath of lead or polythene over a paper or cotton whipping completes the cable.

Various weights of conductor may be used according to the intended use, but 10 lb or 20 lb per mile is typical for junction cables. A 1,000 pair paper-insulated lead-sheathed cable would be about 2¾ in. diameter overall. The characteristic impedance of the pair (unloaded) varies

considerably with frequency in the audio frequency range, from 600–800 ohms at low frequencies, to 200–300 ohms at 3 kc/s. Quad-type cables used in the carrier frequency range have impedances of about 140 ohms.

Star-quad cables are used for providing audio-frequency junctions between exchanges. Special types having superior high-frequency characteristics are used for multi-channel carrier systems.

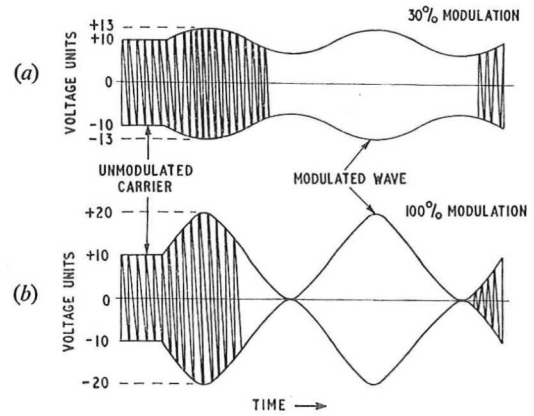
(*b*) For details of the construction of a multi-tube coaxial cable see A.9, Radio and Line Transmission B, 1961, *Supplement*, Vol. 55. p. 19, July 1962.

Cables of this type are used for large-capacity multi-channel carrier systems, up to 2,700 channels, or for television transmission. The characteristic impedance of such cables is in the order of 75 ohms.

***Q*. 10.** *Using squared paper and a suitable scale, draw sketches showing oscillograms of a carrier wave amplitude-modulated by a low-frequency sine wave, when the depth of modulation is* (*a*) 30 *per cent* (*b*) 100 *per cent. State, with explanations, whether the oscilloscope timebase would be synchronized to the carrier or to the modulation frequency.*

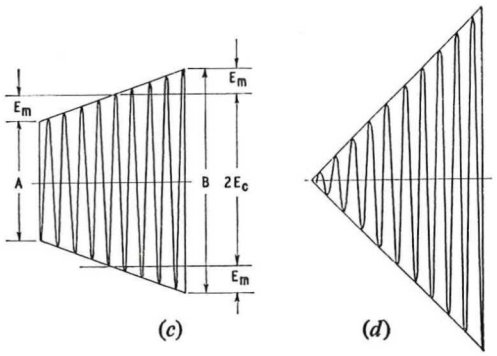
*Explain with the aid of a diagram the display that results from connecting the modulating voltage to the X plates and the modulated voltage to the Y plates. How may this display be used to measure depth of modulation?*

**A. 10.** Sketches (*a*) and (*b*) show amplitude modulated waves having depths of modulation of 30 per cent and 100 per cent respectively.




In order to examine the modulation envelope the oscilloscope timebase should be synchronized to the modulation frequency. When this is done successive sweeps will overlie one another with corresponding points on the modulation envelope superimposed and the display will have the appearance shown in the sketches.

If the modulating frequency alone is applied to the X plates a horizontal line results. Similarly, if the modulated wave is applied to the Y plates a vertical line results. When both are applied simultaneously the vertical line will move from left to right across the screen, growing in amplitude as it moves, so giving the trapezoidal display shown in




sketch (*c*). The transition from minimum amplitude to maximum amplitude is linear if the modulation is distortionless. Sketch (*d*) shows the appearance when the depth of modulation is 100 per cent.

If $E_c$ is the unmodulated carrier amplitude and $E_m$ is the modulating signal amplitude, the trough in the modulated wave has an amplitude $E_c - E_m$ and the peak is $E_c + E_m$. Since the trough and peak of the modulated wave correspond to the portions $A$ and $B$ of the trapezium, $A = 2\ (E_c - E_m)$ and $B = 2\ (E_c + E_m)$ so that $B + A = 4\ E_c$ and $B - A = 4\ E_m$.

$$\text{Thus, } \frac{B - A}{B + A} = \frac{E_m}{E_c}.$$

But $E_m/E_c$ is, by definition, the depth of modulation, so that the measurement may be made by noting the lengths of the appropriate portions of the trapezium displayed and substituting the values in the following expression:

$$\text{Depth of modulation} = \frac{B - A}{B + A} \times 100 \text{ per cent.}$$

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## TELEPHONY B, 1964

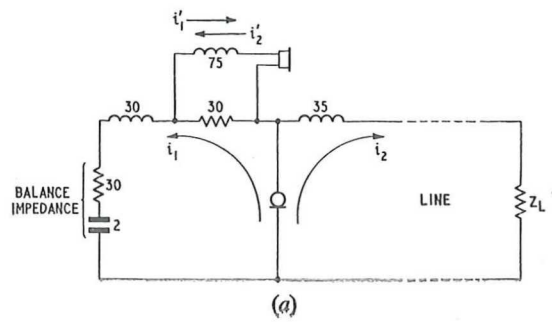
**Students were expected to attempt not more than any six questions.**

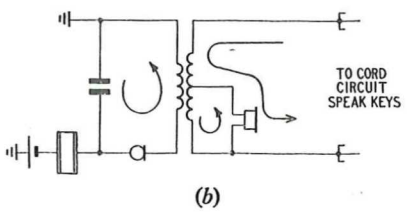
***Q*. 1.** *What do you understand by the term "sidetone" in a telephone? Explain why it is desirable that a telephone should incorporate a means of reducing sidetone, and give simplified circuit sketches to show how this reduction is achieved in:*

(*a*) *a subscriber's telephone, and*

(*b*) *an operator's telephone.*

**A. 1.** Sidetone in a telephone instrument is the reproduction in the receiver of sounds picked up by the transmitter of that telephone. These sounds include both the speaker's voice and room noises.




Sidetone impairs the efficiency of a telephone because (*a*) received speech may be masked by room noise picked up by the transmitter of the listener's telephone, and (*b*) transmitted speech heard at a high level by the speaker in his own receiver causes him to lower the level of his voice, with consequent reduction in the level of speech transmitted to line. A means of reducing these effects is therefore desirable.

Methods of achieving sidetone reduction in a subscriber's telephone and in an operator's telephone are shown in sketches (*a*) and (*b*), respectively.

***Q*. 2.** *Define the terms:*

(*a*) *busy-hour calling rate, and*

(*b*) *unit call.*

*At a central-battery manual exchange having 6,000 exchange connexions, the busy-hour calling rate is 0·8. One-third of the originated calls are local calls (i.e., calls to subscribers on the same exchange) of 3 minutes average duration. Considering the busy-hour conditions, find:*

*(i) the total number of local calls originated,*

*(ii) the number of A-position operators required to handle the local calls, and*

*(iii) the average number of cord circuits simultaneously in use on local calls.*

**A. 2.** (i) 1,600 calls, (ii) 8 operators, (iii) 80 cord circuits.

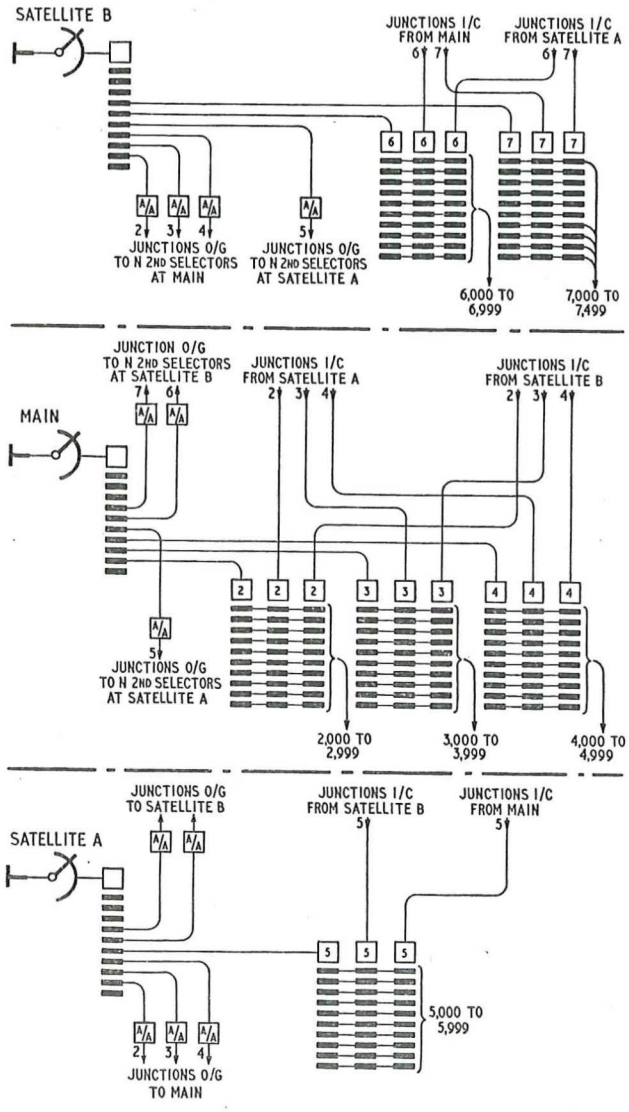
***Q.* 3.** *The numbering scheme of a non-director area is as follows:*

| | |
|---|---|
| *Main exchange* .. .. .. | 2,000 to 4,999 |
| *Satellite A* .. .. .. | 5,000 to 5,999 |
| *Satellite B* .. .. .. | 6,000 to 7,499 |

*Draw a simplified trunking diagram for the above area and state the advantages of the method of trunking you illustrate.*

*What advantage could be gained by giving 5-digit numbers to subscribers on satellite B?*

**A. 3.** The sketch shows a simplified trunking diagram for the area using group-selector type satellite exchanges. This method of working tends to be costly in terms of junction line plant, because direct junctions have to be provided between all three exchanges.


SATELLITE B
JUNCTIONS I/C FROM MAIN
JUNCTIONS I/C FROM SATELLITE A
JUNCTIONS O/G TO N 2ND SELECTORS AT MAIN
JUNCTIONS O/G TO N 2ND SELECTORS AT SATELLITE A
6,000 TO 6,999
7,000 TO 7,499
MAIN
JUNCTION O/G TO N 2ND SELECTORS AT SATELLITE B
JUNCTIONS I/C FROM SATELLITE A
JUNCTIONS I/C FROM SATELLITE B
JUNCTIONS O/G TO N 2ND SELECTORS AT SATELLITE A
2,000 TO 2,999
3,000 TO 3,999
4,000 TO 4,999
SATELLITE A
JUNCTIONS O/G TO SATELLITE B
JUNCTIONS I/C FROM SATELLITE B
JUNCTIONS I/C FROM MAIN
JUNCTIONS O/G TO MAIN
5,000 TO 5,999

Moreover the junctions carrying traffic to the main exchange have to be provided as three separate groups. Similarly the junctions carrying traffic to satellite B are in two groups.

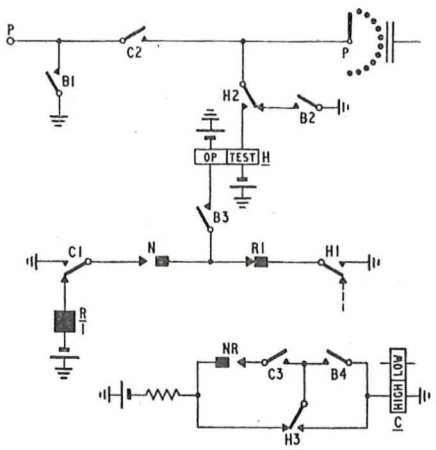
The advantages of group-selector working are to be found in overall simplification and standardization of switching equipment which tends to reduce equipment costs and produces savings in staff training and maintenance effort.

If satellite B subscribers were given 5-digit numbers then the numbering range for that satellite could be 60,000 to 61,500. The two separate junction groups from the main to satellite B would then be replaced by a single group. This would also apply to the junction route from satellite A to satellite B, so that fewer junctions would be needed to carry the same amount of traffic. An extra rank of group selectors in satellite B would off-set some of the saving in junction costs.

***Q.* 4.** *Sketch and describe those circuit elements concerned in the rotary hunting of an earth-testing group selector.*

*Explain why there is an unguarded interval when switching to a disengaged outlet and state its approximate duration.*

**A. 4.** The rotary-hunting circuit elements of a 100-outlet earth-testing group selector are shown in the sketch.




During the vertical stepping, relay C is held operated and when the off-normal contacts, N, close at the first vertical step, an earth is extended via contact C1 to operate relay H. Contact H1 then closes an alternative holding path for relay H via the rotary interrupter contacts, R1. At the end of vertical stepping, relay C releases and contact C1 completes a self-drive circuit for the rotary magnet, dependent on the earth from contact H1. At each step of the rotary magnet, the R1 contacts open to produce the self-drive action and also to disconnect the operating winding of relay H for a short period. During this short period when the R1 contact is open, the H-relay holding path is via contact H2 to the P-wiper. If this wiper is encountering an engaged outlet, the earth guarding that outlet will hold relay H over the "test" winding whilst the R1 contacts disconnect the "operate" winding. It should be noted that the R1 contacts open towards the end of the armature stroke after the wipers have made connexion to a bank contact.

As soon as the wipers are stepped to a disengaged outlet the absence of a guarding earth on the P-wire allows relay H to release and to cut the rotary drive circuit at contact H1 before the rotary magnet can take another step. At the same time contact H2 applies an immediate guarding earth to the outlet to prevent its seizure by another group selector.

Contact H3 reoperates relay C, which at contact C1 reoperates relay H over its "operate" winding. Contact C1 also disconnects the rotary-magnet circuit to prevent further stepping when contact H1 closes again. Contact H2 holds relay H to the P-wire, which at this stage is guarded by contact C2 extending the B1 earth to the P-wiper. Subsequently the outlet is guarded by the earth returned from the P-wire of the selector that has been seized.

Other contacts of relay C disconnect the group-selector A relay and extend the — and + wires to the next switching stage. Release of the A relay causes relay B to release, but relays C and H remain held for the duration of the call.

Since there is a very short interval between the P-wiper encountering a disengaged outlet, and the busying of that outlet by the release of the H relay, there must be an unguarded interval during which there is a risk that two group selectors might seize the same outlet and cause a "double connexion". This unguarded interval persists for only 2 or

3 milliseconds, slightly less than the release time of the H relay, since the H relay of the second selector to encounter the outlet would be held by the H2 guarding earth of the first selector if the difference between the two "encounter times" exceeded this figure.

**Q. 5.** *State the facilities provided by an auto-to-auto relay set suitable for use at a non-director main exchange on a junction to a satellite exchange.*

*Sketch and describe the circuit element concerned in providing "two-stage drop-back" at the end of a dial-pulse train. What advantage is obtained by the "two-stage drop-back" facility?*

**A. 5.** The principal facilities provided are as follows:

(*a*) guarding and holding preceding selectors under the control of the calling subscriber,

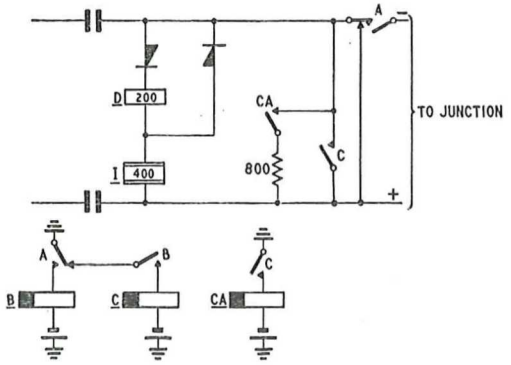
(*b*) guarding and holding the two-wire junction against intrusion,

(*c*) repeating the calling subscriber's dial pulses to the selector at the distant end of the junction,

(*d*) providing transmitter feeding current for the calling subscriber's telephone,

(*e*) applying positive battery to the P-wire to register the call when the called party answers, and

(*f*) repeating back to an operator the supervisory signals from the called party.




Referring to the sketch, relays B, C and CA are operated whilst the A-relay contact is repeating a pulse-train over the junction. At the end of this pulse-train relays A and B remain operated and relay C releases slowly followed by the slow release of relay CA. The transition from the zero-resistance pulsing loop to the 400-ohm I relay holding loop is made in two stages because an 800-ohm shunt is maintained across the I relay during the release time of relay CA. Hence the term "two-stage drop-back".

If the transition were made in one stage, (i.e. without the assistance of the CA relay) the holding path for the A relay in the distant exchange would be transferred to the D and I relay loop at a time when the I relay is not fluxed and hence presents its maximum inductance. This inductance would cause the holding current for the A relay to fall suddenly to zero and then to rise slowly, with the consequent risk of generating a false pulse. The risk would be greatest with a high-resistance junction. A two-stage drop-back arrangement prevents the generation of a false pulse by providing a safe non-inductive holding path for the distant A relay while the current is building up in the I relay.

**Q. 6.** *Draw the cord-circuit of a sleeve-control auto-manual switchboard, and explain the functions of the relay and keys.*

*What purpose is served by increasing the resistance in the sleeve circuit when the speak key is operated?*

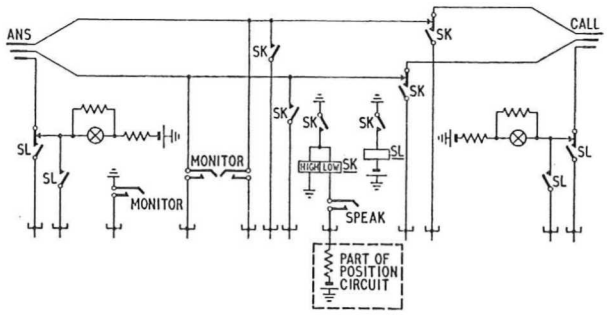
**A. 6.** The required circuit is shown in the sketch. The functions of the keys and relays are as follows:

(*a*) *Speak key*

This key permits the operator to connect her position telephone circuit to both answering and calling cords. It does this by operating relay SK.

(*b*) *SK relay*

The contacts of this relay will normally act as slave contacts of the speak key to provide the facility described in (*a*) above, but this relay serves also to prevent a number of cord circuits from being connected to the position telephone at one time. Only one speak key is allowed to be effective at any one time, since the first SK relay to be operated will connect its low resistance winding to the common resistance battery supply in the position circuit, and if any other speak key is then operated its associated SK relay will receive insufficient current to operate.




The contacts of the SK relay actually divert the tip and ring wires of both answering and calling cords via a series chain of key and relay contacts in the position circuit so that dialling, ringing etc. may be performed separately on either cord as required.

(*c*) *SL relay*

This is virtually a slave of the SK relay and its contacts are used to divert the sleeve wire of both answering and calling cords via the position circuit. This provides the change of sleeve-circuit resistance referred to in the last part of the question.

(*d*) *Monitor key*

This key permits the operator to monitor (listen only) on the cord circuit. The earth contact of the monitor key controls a relay in the position circuit which connects the telephone receiver to the other contacts of the monitor key via an impedance-changing transformer, which reduces the insertion loss whilst monitoring a conversation.

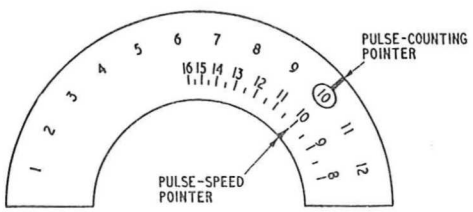
Whereas, in a bridge-control system, operation of a cord-circuit speak key causes an "operator answer" signal to be applied to one or both wires of the speaking path, in the sleeve control system this signal is given over the sleeve conductor. When the SL contacts are operated the resistance of the sleeve circuit is increased and this unbalances the circuit of a differentially-connected relay, DR, in the sleeve circuit of the line termination. Contacts of the DR relay then apply appropriate signals to the line.

**Q. 7.** *Describe a device for measuring the pulse repetition rate (pulse speed) of a telephone dial. Explain carefully the principles employed in the measuring device.*

*Sketch a circuit element to show how a* 0–100 *mA meter may be used to measure the make-to-break ratio produced by a pair of contacts on a pulse-generating machine.*

**A. 7.** The device to be described, known in the British Post Office as a Tester No. 43, is fitted on test desks and is used in association with the test cord. Pulses from the dial under test are repeated by a relay and each dial break period is extended as a make pulse to the tester's electro-magnet.

The tester consists of a pulse-counting mechanism and a clock-work escapement (i.e. a time-measuring mechanism). For pulse counting the electromagnet drives a ratchet and pawl to rotate a pointer, in steps, over a linear scale marked as in sketch (*a*). This part of the




(*a*)

device records only the *number* of pulses delivered by the dial under test.

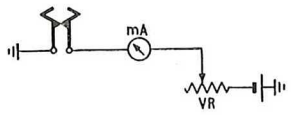
The pulse-counting mechanism is also used to control the escapement, in that it causes the escapement to start immediately the first pulse is received and to stop immediately the tenth pulse is received.

Thus the escapement records the time taken by the dial to deliver 10 pulses. This elapsed time is displayed on a pointer, which moves concentrically with the pulse-counting pointer. If the dial under test delivers pulses at exactly 10 pulses per second then both pointers move more or less together at the same average speed. After each test has been made both pointers have to be returned to the zero position by turning an external lever, which also rewinds the spring controlling the escapement. If the number of pulses received from a dial is greater, or less, than 10 the escapement is not arrested and no measurement of pulse speed is made.

Now if $N$ is the number of pulses delivered by a dial in one second, and if $T$ is the time in seconds required by the dial to deliver 10 pulses,

$$\text{then } T = 10 \times \frac{1}{N}, \text{ or } N = \frac{10}{T}.$$

Hence $N$, which is by definition the pulse repetition rate measured in pulses per second, is shown to be inversely proportional to the time $T$, which is measured as the distance moved by the escapement, or time, pointer. This gives the reason for the non-linearity of the inner scale and explains how the tester, which actually measures the time taken by the dial to deliver exactly 10 pulses, is able to display this as the dial-pulse speed.




(*b*)

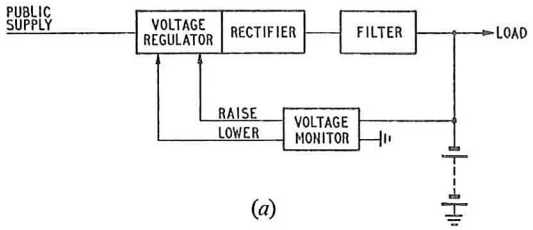
Sketch (*b*) shows how a current-measuring meter may be used to measure the percentage make ratio of a pair of pulsing contacts. For a steady reading a heavily-damped movement is required. If the meter measures 100 mA at, or near, full-scale deflexion, and if the variable resistor VR is set to give this value when the contacts are short-circuited, the meter will give a direct reading of percentage make ratio, i.e.

$$\frac{\text{make period}}{\text{make period} + \text{break period}} \times 100 \text{ per cent.}$$

***Q.* 8.** *Describe with the aid of sketches a means of controlling the output voltage from a floated-battery power plant at a medium-size 50-volt automatic telephone exchange.*

*Explain why it is necessary to prevent the exchange voltage from falling too low, or rising too high.*

**A. 8.** In a modern high-efficiency floated-battery power plant using semiconductor rectifiers, the d.c. output voltage from the rectifier is regulated by varying the alternating voltage fed to the input of the rectifier as shown in sketch (*a*). Typically the regulator may be a motor-driven variable-ratio auto-transformer. Sketch (*b*) shows the voltage

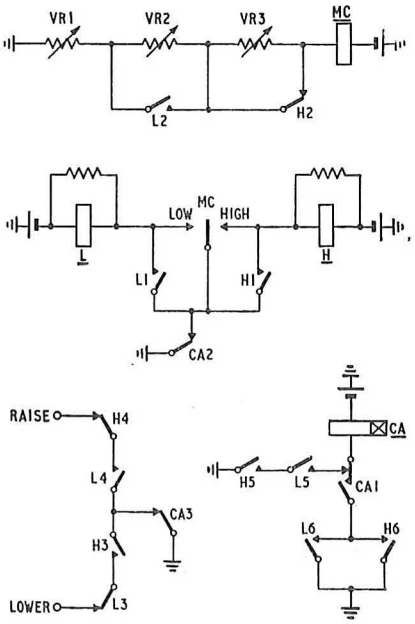



(*a*)

monitor relay MC and the associated relays through which it controls the voltage regulator via the "raise" and "lower" leads.

Relay MC is set to its neutral position by adjusting VR1. Thereafter a change of ±0·3 volts across the exchange load will change the current through relay MC sufficiently to cause it to close its "high" or "low" contact as appropriate. Assuming the voltage rises sufficiently for relay MC to close its "high" contact, the relief relay H will operate and will lock at contact H1. The H2 contact now opens and causes the current in MC to fall, VR3 being pre-set so that this fall of current in relay MC resembles a bias of −0·5 volts which is sufficient to cause the MC contact to leave the "high" position, but is insufficient to cause it to close the "low" contact. With relay H operated the H3 contact drives the regulator motor in a direction that will lower the a.c. input to the rectifier. When the rectifier output has fallen by 0·1 volts relay MC closes its "low" contact and operates relay L. Contact L3 stops the regulator motor to prevent further lowering of the output voltage, but as contact H4 is still operated the L4 contact does not apply a control signal to the "raise" lead.

With contacts H5 and L5 both operated, relay CA operates to release the L and H relays at contact CA2. Relay CA locks at contact CA1 until both L and H relays have released and contact CA3 disconnects the earth from the control leads to prevent a false control signal during any overlap between the release times of relays L and H. After its slow-to-release period, relay CA reconnects the earth to the control relays at contact CA2, and to the control leads at contact CA3.




(*b*)

If the voltage across the load falls, the circuit operation is similar except that relay L operates first to give a "raise" signal at contact L4 and relay H operates subsequently to disconnect the control signal at contact H4. Both sequences achieve a change of 0·1 volts, either down or up, in the voltage across the exchange bus-bar.

The relays and selector magnets of the automatic exchange equipment are affected by the exchange voltage in that a voltage increase shortens their operate times and lengthens their release times, whilst a voltage reduction has the opposite effect. The various items are not all affected to the same extent by a given change in voltage and if this change becomes excessive the timing sequences, upon which successful operation depends, would be endangered. In extreme conditions relays and selector magnets would fail to operate with an excessively low voltage, whilst an excessively high voltage would increase power dissipations and introduce fire risks.

The equipment is normally designed to perform satisfactorily between 46 volts and 52 volts.

***Q.* 9.** *A large automatic exchange without a local auto-manual switchboard is to be accommodated in a building having a ground floor and one upper floor.*

*Sketch a suitable equipment layout and indicate the reasons for choosing the layout you adopt.*

*What considerations determine the quantities of the following equipments provided at a new automatic exchange:*

(*a*) *subscribers' calling equipments, and*

(*b*) *final selectors?*

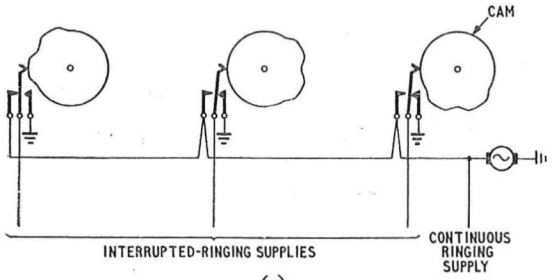
***Q.* 10.** *Briefly describe the ringing-current generation and distribution arrangements at a large automatic exchange. Illustrate your answer by means of sketches.*

*How would you estimate the power output required from the ringing generator?*

**A. 10.** Ringing current is a 25 c/s (previously 16⅔ c/s) a.c. supply at about 75 volts r.m.s., and at a large exchange is produced by a motor-generator set known as a "ringer". Two ringers are normally provided, the regular one driven from the public supply mains and the reserve from the exchange battery, but modern practice favours two identical ringers, both battery-driven. Battery-driven ringers are of the dynamotor type with the motor and generator windings on the same armature, the a.c. being collected by carbon brushes in contact with slip rings.

## TELEPHONY B, 1964 (*continued*)

An extension of the rotor shaft drives reduction gearing to revolve a set of three cams at 20 rev/min. Sketch (*a*) shows how these three cams, spaced 120° apart, divide the continuous-ringing supply into




(*a*)

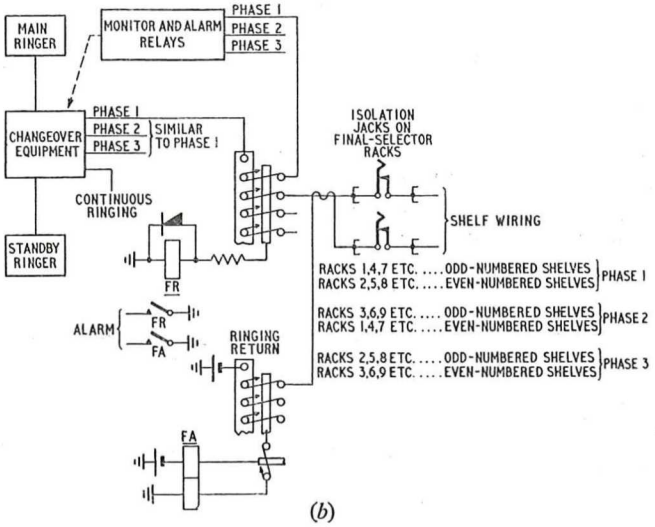
three sequential interrupted-ringing phases, each having the same characteristic of an active period of 0·4 seconds on, 0·2 seconds off and 0·4 seconds on, followed by a 2-second silent period. At any instant only one phase is active and two are silent and by spreading the exchange ringing load over the 3-second cycle a ringer of lower power output rating may be used. The load on the continuous-ringing supply, which is used for miscellaneous manual ringing purposes, is relatively small.

Sketch (*b*) shows the fusing and distribution arrangements, including a systematic method of allocating the three phases to the shelves of each group of three final-selector racks. The interrupted and continuous-ringing primary supplies are continuously monitored by a group of relays and if any of the four supplies fails the exchange load is automatically switched over to the standby ringer and the appropriate alarms are given.

The power output $P$ required from an exchange ringer may be determined from the traffic data as follows:

Let $E$ be the traffic in erlangs flowing to the subscribers and $T$ the average call holding time.

Let $t$ be the average time to answer, $N$ the number of lines simultaneously receiving an active ring and let $W$ be the power that must be supplied to a final selector to operate a subscriber's bell or calling device.




(*b*)

Now the number of final selectors simultaneously in use $= E$, and the number of final selectors simultaneously calling $= E \times \frac{t}{T}$.

At any instant only one third of these final selectors are in the active ringing condition.

$$\therefore N = \frac{E}{3} \times \frac{t}{T} \quad \text{and P} = \text{NW}$$

$$\therefore P = \frac{E}{3} \times \frac{t}{T} \times W.$$

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## TELEGRAPHY B, 1964

**Students were expected to attempt not more than any six questions.**

**Q. 1.** *Sketch graphs showing the change in current with time when a p.d. is applied across and then short-circuited across (a) a resistance in series with a capacitance and (b) a resistance in series with an inductance.*

*What is the significance of these graphs in relation to the transmission of telegraph signals by direct currents?*

**Q. 2.** *A telegraph channel is designed for the transmission of signals at a modulation rate of 50 bauds. Calculate the operational speeds, in words per minute, if the channel were used for the following: (a) morse code, (b) three-condition cable code, (c) five-unit code, synchronous, (d) five-unit code, start-stop. Indicate what assumptions are made in your calculations.*

**A. 2.** A modulation rate of 50 bauds indicates that the duration of a unit, or shortest signal element, is $\frac{1}{50}$ second, hence in a minute, which is the usual period to which the operational speed is related, there can be a maximum of $50 \times 60 = 3{,}000$ unit signal elements.

Since different codes require varying numbers of signal elements to transmit the same information, certain assumptions need to be made to provide a common basis. A general assumption is that in plain language one word comprises five letters and one word-space. Other assumptions, if any, are given with the individual codes.

(*a*) *Morse code.*

The basic element of this code is the dot of one unit length, each character comprising a combination of dots and dashes, a dash being equal in length to three dots. The elements of a character are separated by a space of one dot duration: between letters there are three and between words seven dot spaces. On average it is assumed that there are nine dot elements per character (including letter-space) although this varies with the type of traffic, hence an average number of unit elements in a word will be $(5 \times 9) + 7 = 52$ elements. At the 50-baud modulation rate the operational speed will be $\frac{3{,}000}{52} =$ 57·7 words per minute.

(*b*) *Three-condition cable code.*

This is a more efficient form of morse code signalling, used on d.c. submarine telegraph cables, in which the dot and dash conditions are of equal length and are distinguished by change of polarity, the space conditions being represented by earth. No space is required between elements, one element duration of earth is used for letter spacing and three elements duration for word space. The number of elements per word is considerably reduced and an average figure taken is $(3{\cdot}7 \times 5) + 3 = 21{\cdot}5$.

Hence the operational speed at a modulation rate of 50 bauds will be $\frac{3{,}000}{21{\cdot}5} =$ 140 words per minute.

An assumption made here is that the elements are received fully formed. In practice "curbing", or shortening of the time for which either polarity is applied, is usually introduced to assist in signal shaping. This would affect the operational speed if the shortened periods of polarity are to be related to the $\frac{1}{50}$ second period.

(*c*) *Five unit code, synchronous.*

Each character is of the same length, i.e. five elements, and a word space is of one character length, hence the average word requires $5 \times 6 = 30$ unit element durations.

Therefore, operational speed $= \frac{3{,}000}{30} =$ 100 words per minute.

(*d*) *Five unit code, start stop.*

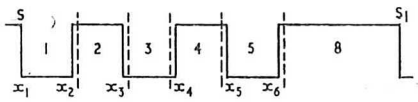
Each character is of the same length, $7\frac{1}{2}$ elements, and a word space is one character, hence one word is $7\frac{1}{2} \times 6 = 45$ elements.

Hence operational speed $= \dfrac{3{,}000}{45} = \underline{66{\cdot}6 \text{ words per minute.}}$

**Q. 3.** *Give sketches to show how signals with* 10 *per cent telegraph distortion would appear upon the scales of* (*a*) *a start-stop distortion meter, and* (*b*) *a synchronous* (*isochronous*) *distortion meter. For each meter show on a timing chart for any typical five-unit signal how the characteristic instants* (*i.e. transitions*) *are related to the meter readings.*

*If measurement on a voice-frequency channel gave either of these readings, would you regard the channel performance as satisfactory or not? Give reasons for your answer.*

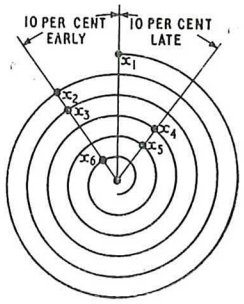
**A. 3.** (*a*) Sketch (*a*) shows a 5-unit signal with start and stop elements added. Each transition in this signal has a 10 per cent distortion, which indicates that the transitions are displaced from the ideal instants indicated by the dotted lines. The ideal instants, for a 50-baud




(*a*)

modulation rate, occur at intervals of 20 milliseconds, or multiples thereof, after the front edge of the start element indicated at S. A transition with 10 per cent distortion means that the time at which it occurred is displaced by 10 per cent of 20 milliseconds from the time at which it was expected to occur.

Sketch (*b*) shows the display that this signal would give on the cathode-ray tube of one form of start-stop telegraph distortion measuring set. Each transition causes a brightening of a spiral trace, which

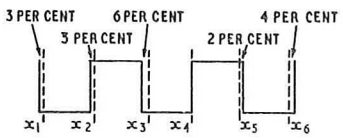



(*b*)

begins at $x_1$, when the signal is applied, the time of each of the six convolutions of the spiral being made exactly equal to a nominal signal interval, i.e. 20 milliseconds. If the signal were without distortion then the transition indications would appear in a vertical line.

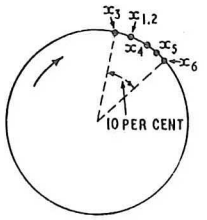
Referring to sketch (*a*) it will be noted that, although elements 2 and 4 are exactly 20 milliseconds in duration, in the start-stop mode of measurement in which the timing is referred to point S, the associated transitions have a distortion of 10 per cent. At the same time, although element 3 is lengthened by 20 per cent and element 5 shortened by the same amount, the distortion indicated is 10 per cent. Point S1 indicates the beginning of the next start element which initiates a new timing cycle.

(*b*) In measuring the isochronous distortion of a train of pulses, each transition is arranged to form a bright spot on a circular trace of a cathode-ray oscilloscope. The time of revolution of the trace is adjusted to be equal to the mean element period of the applied signal.




(*c*)

In practice this is effected by varying the rate of revolution until the distortion display as a whole remains stationary. Sketch (*c*) represents a distorted train of telegraph reversals in which the nominal element period is 20 milliseconds. If the signal were without distortion then the brightened spots would occur at the same place on the circular trace at an arbitrary point equivalent to the dotted lines in sketch (*c*). The display that would arise from the given train of pulses is shown in sketch (*d*), assuming the trace to be revolving in a clockwise direction.




(*d*)

The persistence of the glow on the oscilloscope enables a number of successive spots to be seen together and in a given observation the isochronous distortion is assessed as the sum of the time that the earliest transition is early and the latest transition is late expressed as a percentage of a unit element. In the example shown $x_3$ is 6 per cent early and $x_6$ is 4 per cent late. The isochronous distortion is therefore 10 per cent.

The limit which the British Post Office imposes on telegraph distortion for a single v.f. telegraph channel is 10 per cent for the isochronous method and 8 per cent for the start-stop method of measuring. The distortion shown in sketch (*b*) is therefore excessive but that of sketch (*d*) is just acceptable. It is essential that a low figure of distortion on a single v.f. telegraph channel should not be exceeded because of the inevitability, on a switched network, that several channels will be connected in tandem (series). In the worst case, when five channels are so connected, it is necessary that a total start-stop distortion of 30 per cent is not exceeded. This is likely to be achieved only if the start-stop distortion of a single channel does not exceed 8 per cent.

**Q. 4.** *Explain clearly the basic difference between printing-telegraph systems operated on* (*a*) *a synchronous system and* (*b*) *a start-stop system. For what purposes would each system be used and what particular advantages would be gained?*

**A. 4.** (*a*) *Synchronous system.* The transmitter and receiver of a synchronous printing-telegraph system run continuously and the receiver is arranged to keep in step, or synchronism, with the transmitter, This is achieved, generally, by comparing the transitions of the incoming signal with the local timing of the receiver and automatically adjusting the latter as required. Hence, after initial coincidence of phase has been achieved, the receiver will keep in synchronism with the transmitter. Signals must be transmitted continuously and, during periods of no traffic, special idle signals are introduced.

(*b*) *Start-stop system.* The transmitter and receiver run only when a character is being transmitted and received. Each transmitted character is preceded by a start element and followed by a stop element, these two elements being required to start and stop the receiving equipment for each character. During a character period the receiver is kept in step with the transmitter by initial speed adjustments which should be re-checked periodically.

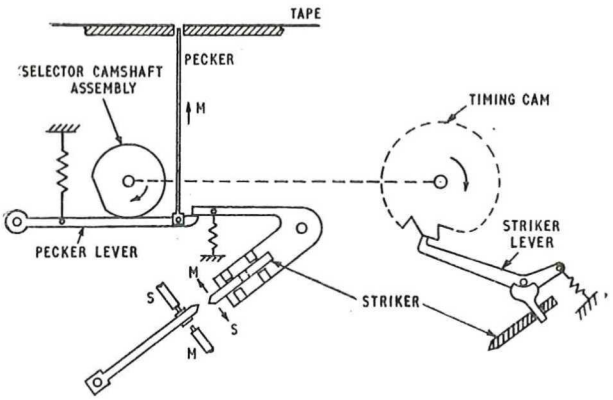
As a synchronous system does not require start or stop elements there can be a reduction in character length and hence an increase in the character clearance rate without any corresponding decrease in element length. Over a v.f. system this will provide an increase in the character clearance rate without any increase in the bandwidth required. A synchronous system is particularly useful for use on time-division multiplex equipment. Such equipment offers the advantage of carrying two or more telegraph channels over a circuit designed to carry a lesser number of non-synchronous channels. A synchronous system can tolerate approximately twice the degree of telegraph distortion permissible in a start-stop system.

The start-stop system has the advantage of simplicity. Also, after line interruptions, the equipments can resume traffic without the necessity of rephasing or other adjustments. This system is widely used for leased, public and telex circuits where straightforward, rather than multiplex working is operative.

**Q. 5.** *Describe, with a sketch, the general principle of operation of an automatic transmitter suitable for five-unit code. What provision is made for avoiding the generation of mutilated signals at the instants of starting and stopping the machine?*

**A. 5.** An automatic transmitter translates perforations in a paper tape into five-unit start-stop signals suitable for teleprinter operation.

Each character on the tape is represented by a combination of holes in five possible positions across the width of the tape. These are read by a series of light rods (peckers) which are allowed to move upwards in turn under the action of light springs. If a hole is present in the tape the pecker rises fully to the marking position; if there is no hole the blank paper is sufficiently strong to hold the pecker down against its spring in the spacing position.




The sketch shows one typical pecker and its pecker lever. When the transmitter head is clutched to the motor drive the selector-cam assembly rotates. The cam assembly comprises a start-stop cam (not shown), five pecker cams and a timing cam (shown to one side for convenience). The start-stop cam operates a start-stop lever (not shown) which is similar to a pecker lever without a pecker. The initial rotation of the cams causes the start-stop lever to ride downwards so that the common lever under control of its springs moves to the spacing (S) position. Rotation of the timing cam brings the striker lever to the first "drop-off" position shortly after the movement of the common lever into "space".

The common lever carries a striker with a knife-edge mounted opposite to the knife-edge of the transmitter tongue. When the striker lever operates, the striker is moved towards the tongue, displacing it from "mark" to "space" for the commencement of the start element of the transmitted signal. Subsequently, the striker lever is withdrawn in readiness for the next strike.

As the transmitter cam continues to rotate, the pecker levers are allowed to move upwards in turn, if their peckers permit. Depending on the presence or absence of perforations, the common lever either moves to mark or remains on space. Following each pecker operation the timing cam causes the striker lever to actuate the striker and thus to move the transmitter tongue between its contacts in accordance with the holes in the tape. A spring-loaded roller, pressing on the knife-edge of the tongue (not shown), ensures that it remains pressed against whichever contact it is placed upon until moved by the striker to the opposite contact. The accurate cutting of the timing cam ensures that the signal transitions are correctly timed relative to the beginning of the start signal.

Following the five pecker operations the start-stop lever rides upwards, moving the common lever to "mark", and the final "drop off" of the timing cam causes the striker to set the transmitter tongue to "mark" for the stop signal element. Before the rotation of the cams is complete a tape-feed mechanism (not shown) moves the perforated tape forward one pitch ready for the next character.

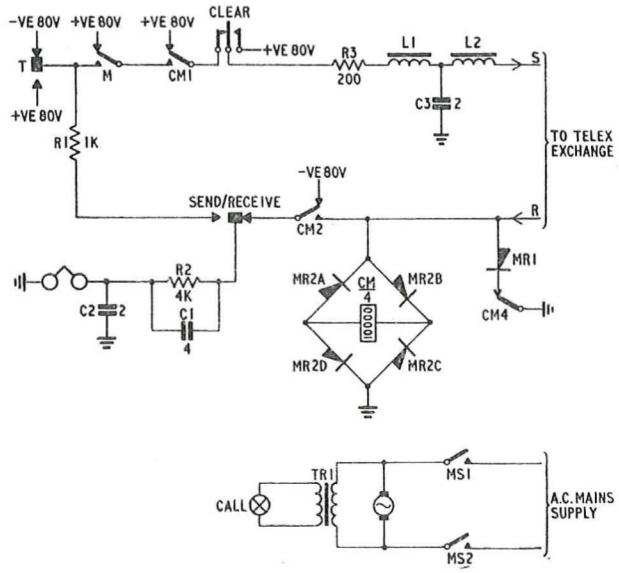
The auto-transmitter is operated by a motor which runs continuously while messages are being sent. The cam assembly of the auto head is coupled to the motor drive-shaft by a clutch operated by a manual control lever. When this lever is operated the cams rotate continuously and signals corresponding to the tape perforations are transmitted continuously without any pause between characters. Restoration of the control lever moves an abutment into the path of the clutch detent so that at the end of the current transmitter cycle, when the cam has reached its rest position, the clutch is decoupled and transmission ceases. As decoupling occurs during the stop signal element, mutilation of signals cannot occur.

***Q.* 6.** *What are the relative advantages of the following methods of transmission on submarine telegraph cables: (a) double-current morse code, (b) three-condition cable code, (c) double-current five-unit code. For each code sketch the waveform of a typical generated signal.*

*For each of the above systems, what equipment would be needed at the receiving office to reproduce the received messages in the form of printed characters?*

***Q.* 7.** *Sketch those circuit elements of a subscriber's station equipment connected to a teleprinter automatic exchange which are required to respond to an incoming call. Describe the circuit operation for an incoming call from the commencement of the calling signal to the completion of the clearing signal.*

**A. 7.** The sketch shows the circuit elements of a telex subscriber's station equipment concerned with an incoming call. When the station is in the free-line condition, start polarity is extended to the telex




exchange over the S-wire from contact CM1. The telex exchange returns start polarity over the R-wire via the forward-biased rectifier MR1 and CM4 to earth, thereby short-circuiting relay CM connected in the full-wave rectifier bridge MR2.

The commencement of an incoming call is signalled to the station when the telex exchange connects the R-wire to stop polarity. This back-biases rectifier MR1 and consequently removes the short-circuit from relay CM, which operates to the stop polarity on the R-wire. Contact CM1 connects the S-wire to start polarity from the teleprinter motor "on-speed" contact, M; CM2 connects the teleprinter receiving electromagnet to the R-wire, CM3 operates relay MS and CM4 disconnects rectifier MR1 from the R-wire. Contacts MS1 and MS2 connect a.c. mains to the teleprinter motor and to transformer TR1 to light the call lamp. When the teleprinter motor reaches the correct speed the teleprinter motor "on-speed" contact M operates, this contact disconnects the start polarity from the S-wire and connects stop polarity from the teleprinter transmitter tongue, T. The start polarity on the S-wire indicates that the teleprinter can now interpret signals correctly. It is detected by the exchange equipment connecting the incoming call, which then transmits figure-shift WRU (secondary of D) to the station over the R-wire.

The character combination WRU is received by the teleprinter electromagnet causing the teleprinter transmitter to transmit an answer-back code to the caller over the S-wire.

The call is now established and communication between the calling and called station is possible on a 2-wire simplex basis with local record. Relay CM remains operated irrespective of the polarity of the R-wire during the call due to the rectifying action of rectifier MR2. The resistance of the winding of relay CM is high to minimize its effects on the signals received by the teleprinter. Signals from the teleprinter (either from the keyboard or the answer-back unit) are transmitted over the S-wire while a leak circuit, via resistor R1 and the send-receive switch of the teleprinter (which operates whenever the transmitter is active), provides a local record of all signals transmitted to the caller. Capacitor C1, with resistor R2 in parallel, ensures that the current in the receiver rises rapidly at the commencement of a transition. Capacitor C2, by absorbing transient currents resulting from the redistribution of flux in the receiver when its armature is operated, prevents these currents from flowing into the R-wire and causing interference to adjacent telephone pairs in the cable.

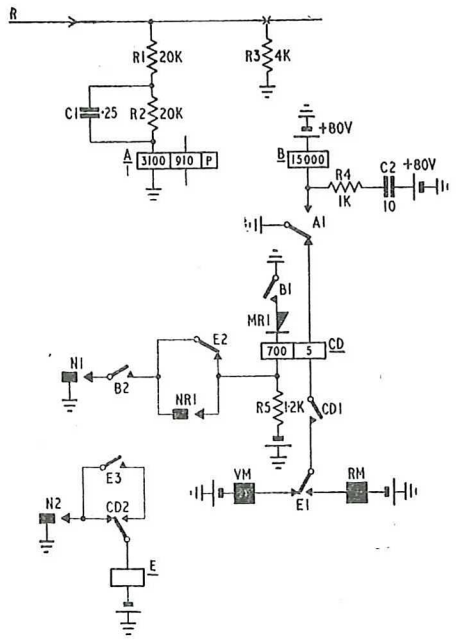
Inductors L1 and L2, with capacitor C3, form a low-pass filter between the transmitter and the S-wire. The filter passes the fundamental frequency of the teleprinter signals (25 c/s at 50 bauds) but stops

harmonics of this fundamantal frequency lying in the speech band, which would otherwise interfere with telephone circuits routed in the same cable on adjacent pairs. Resistor R3, in series with the low-pass filter prevents the filter from oscillating when responding to the square waveform of signals from the transmitter. This would otherwise introduce distortion to the signals when received by high-impedance receiving equipment and also increase interference in telegraph circuits routed in adjacent pairs.

If the calling party clears, the exchange equipment disconnects the R-wire. If the called station clears by operating the clear key, the resultant start polarity on the S-wire, persisting for longer than 300 ms, also causes the exchange equipment to disconnect the R-wire. Disconnexion of the R-wire releases relay CM. Contact CM1 maintains start polarity on the S-wire. Contact CM2 disconnects the R-wire from the receiver and connects the receiver to stop polarity, so preventing the teleprinter from "racing" while the motor is stopping. Contact CM3 releases relay MS and extinguishes the call lamp. Contact CM4 connects rectifier MR1 to the R-wire, so preventing re-operation of relay CM when start polarity is returned by the fully-released exchange equipment.

***Q.* 8.** *Sketch and describe the operation of the circuit elements which control the vertical and rotary stepping of the final selector in a teleprinter automatic exchange. Explain in your description the significance of the make and break periods of the pulses and of the inter-train pause.*

**A. 8.** A final selector is designed to accept two trains of dial pulses. The first train controls the selector vertical electromagnet, to position the wiper carriage vertically, and the second train controls the rotary magnet, positioning the wipers by rotating the wiper shaft to connect the wipers to the selected bank contact. The two trains of dial pulses are received on the forward transmission path of the selector R-wire. When the selector is seized to establish the call, the R-wire is connected to stop polarity (negative), which comes from the calling station. Operation of the calling-station dial causes the stop polarity (make period) to be interrupted a number of times, corresponding to the digit dialled, by pulses of start polarity (positive break period).




Between the two trains of pulses an interval of stop polarity (make period) considerably in excess of the make periods of dial pulses, is obtained by the time taken to wind up the dial for the second train of pulses and a fixed period introduced by the dial while unwinding (lost motion period) prior to sending a dial pulse train. Their sum forms the inter-train pause, which allows the final selector time to change over the circuit so that the second train controls the rotary magnet instead of the vertical magnet. The necessary circuit elements are shown in the sketch.

Relay A, connected to the selector R-wire, repeats the dial pulses to the selector magnets: distortion of the pulse ratio by the relay is kept to a minimum by making relay A a magnetically-polarized relay and by using a signal-shaping network consisting of resistors R1, R2 and capacitor C1.

Relay A is operated by the stop polarity connected to the R-wire during the inter-train pause preceding the first train of pulses to be received by the final selector. Contact A1 operates relay B and contact B1 operates relay CD over its 700-ohm winding while contact B2 prepares to apply a short-circuit to the same winding. The operation of contact CD1 prepares a circuit for the vertical magnet and contact CD2 prevents relay E from operating directly contact N2 operates. Relay A subsequently pulses to the first train of dial pulses by releasing to the break periods and operating to the make periods. For each release of relay A its contact energizes the vertical magnet, causing the wiper carriage to be raised one step. The first vertical step taken by the wiper carriage causes the selector off-normal contacts to operate, contact N1 then short-circuits the 700-ohm winding of relay CD making it slow to release. The release lag is sufficient for relay CD to remain operated for each period that relay A is operated during the dial-pulse train. The flux in the magnetic circuit of relay CD is maintained by current flowing in the 5-ohm winding in series with the vertical magnet during the periods in which relay A is released to the break period of the dial-pulse train. Rectifier MR1 assists in maintaining the level of flux because it is reverse biased to the e.m.f. induced in the 700-ohm winding when current is rising in the 5-ohm winding on completion of the magnet circuit by the release of contact A1. It thereby effectively disconnects the short-circuit on the 700-ohm winding, which would otherwise slow down the rise of flux but when contact A1 operates and disconnects the 5-ohm winding the e.m.f. induced in the 700-ohm winding forward-biases the rectifier, allowing the short-circuit to be fully effective. At the end of the first train of dial pulses, relay A remains operated during the inter-train pause and, because the pause is longer than the release lag of relay CD, relay CD releases. Contact CD2, in restoring, completes a circuit for relay E to operate. The operation of contact E1 disconnects contact A1 from control of the vertical magnet and connects the rotary magnet in its place. Contact E2 disconnects the short-circuit from the 700-ohm winding of relay CD, allowing it to re-operate, and contact E3 provides a holding circuit for relay E when CD2 contact has operated. The operation of contact CD1 prepares a circuit for the rotary magnet and CD2 holds relay E operated via contact E3.

Relay A will then repeat the second train of dial pulses. Each time contact A1 releases, the rotary magnet is energized causing the wipers to be rotated. Directly the wiper carriage takes the first rotary step the rotary off-normal springs, NR, operate, contact NR1 applies a short-circuit to the 700-ohm winding of relay CD making it slow to release. The level of flux in relay CD during the second train of pulses is maintained by current flowing in its 5-ohm winding in series with the rotary magnet and by the action of rectifier MR1. Relay CD therefore remains operated during the rotary train in the same manner as described for the vertical train. At the end of the train, relay CD releases and connects a circuit (not shown in the sketch) for testing whether the selected line is free, busy, or in the absent condition.

Relay B must remain operated during the periods for which relay A is released to the break periods of the dial-pulse train. This is achieved by capacitor C2 in parallel with relay B providing relay B with sufficient release-lag for it to hold.

The release lag of relay B, provided by capacitor C2, is therefore arranged to exceed the longest break period of dial pulses acceptable to the selector electromagnet.

The release lag of relay CD must be sufficient for the relay to hold during the longest make period.

***Q.* 9.** *In what circumstances is the telephone used for (a) acceptance, (b) forwarding, (c) delivery of telegrams? Describe the essential equipment provided at a large telegraph office for handling telegrams by telephone. Describe briefly methods of reducing the delay in answering calls during busy periods.*

**A. 9.** (*a*) A phonogram operator uses a telephone set for the acceptance of telegrams dictated from a subscriber's telephone or from a call office.

(*b*) Telegrams are forwarded by telephone (the telephone-telegram service) between head offices and small branch telegraph offices when the volume of traffic does not justify the normal provision of teleprinters.

(*c*) Delivery of telegrams is generally effected by telephone if the addressee has a telephone, particularly when the address is in the form of a telephone number. If required, written confirmation is sent by post.

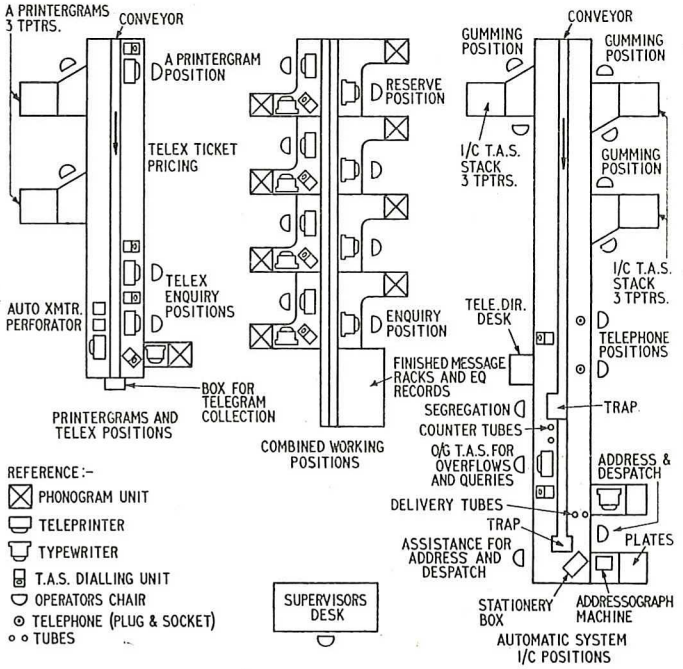
At a large telegraph office combined working is generally employed, whereby each telegraph operator can accept and forward messages. The main equipment provided per operator is a 20 or 40-line phonogram keyboard and an L-shaped table on which is a telegraph typewriter for taking down messages, a tape teleprinter and a dialling unit for

forwarding telegrams over the teleprinter automatic switching network. Visible index or other files for obtaining routing information and telephone directories are at hand and also a serial-number stamping machine. A foot-operated transmitter cut-out switch is provided to eliminate the pick-up of room noise by the operator's lightweight headset.

During busy periods delay in answering calls is reduced in the largest offices by means of an automatic call-queueing system (phonogram automatic distribution), whereby calls are offered, in the order of arrival, to operators as they become free. The length of the queue can be adjusted in units of five by the supervisor up to a maximum determined by the needs of the office. Twenty-five queue places are typical for an office having 50 positions. Callers finding all queue places busy receive "busy" tone.

At other offices where separate or combined working is used, the call lamps are arranged to flash after the lapse of a predetermined interval so that callers who have been waiting a long time can be given early attention. At time of heavy pressure the operators can be allowed to devote themselves to answering calls, leaving the forwarding of messages in abeyance until a slacker period occurs. Alternatively, certain operators can exclusively answer calls, whilst others are occupied only in forwarding messages over the teleprinter network from the same operating positions or from separate teleprinter positions.

**Q. 10.** *Sketch the layout of a telegraph instrument room which includes a total of about twenty teleprinter positions. Describe the possible sources of incoming telegrams, the methods of disposing of the outward telegrams and the procedure for handling telegrams in transit through the instrument room.*




**A. 10.** The sketch shows a possible layout for a telegraph instrument room in which the following methods of telegraph working are used.

(*a*) *Telegrams handed in by members of the public.*

These are brought into the instrument room either by hand or by pneumatic tube from the public office and delivered to operators at the positions in the centre of the room, distribution being under the control of a supervisor who ensures that the work loading is even.

(*b*) *Telegrams telephoned into the instrument room by members of the public.*

These are received on the positions in the centre of the room and are handled by the same operators who handle the traffic mentioned in (*a*) above. The operator receives the call on a telephone headset and, as it is dictated by the subscriber, makes a copy on a typewriter. The telegram can then be sent by the same operator, over the automatic switching system to which the operator's teleprinter is connected.

(*c*) *Telegrams delivered to the instrument room from other instrument rooms connected to the automatic switching network.*

The machines at the right-hand end of the room receive inward telegrams in the form of gummed tape, which is affixed to telegram forms by operators seated by the incoming machines. The telegrams are then placed on the conveyor and, at the end of the table, are sorted for dispatch by one of the methods available, according to the service provided for the addressee. Special addressees may have registered telegraph addresses and these are specially dealt with at the end of the table. Others may have their telegrams delivered over the telex network, or by special private termination (not shown on the diagram) if the volume of traffic justifies such provision. The remainder are sent by pneumatic tube to the delivery room, where they are further sorted for dispatch by postmen or messengers.

(*d*) *Telegrams delivered direct to the instrument room from the Telex network (Printergrams) or possibly direct from a busy subscriber by private wire.*

Printergram messages appear as tape on the machines at the left of the room and are dealt with in much the same way as messages from teleprinters associated with the automatic system referred to in (*c*) above. At the end of the conveyor belt they are sorted, to determine the method of onward despatch, and are handed to whichever operator has to deal with them.

To ensure that no message is lost in its passage through the office each one is given a distinctive code or number, the actual arrangement depending on the office and the traffic quantities that are handled. Throughout the day a continuous check is made to see that the messages are handled in the correct way and that the correct routing has been applied. This is a clerical function and is carried out at special record desks not shown in the sketch.

In order to deal with queries regarding messages, originating offices keep the copies for a predetermined period. Since all messages have a distinctive code and number, it is possible to trace any one by reference back to the originating office, which can, if need be, refer to the filer of the message from the local office references for that particular message.

## MATHEMATICS C, 1964

**Students were expected to attempt not more than any six questions.**

**Q. 1.** (*a*) *Given* $R_0 = \dfrac{R_2(R_1 + 2R_0)}{2R_0 + R_1 + 2R_2} + \dfrac{R_1}{2}$, *express* $R_0$ *in terms of* $R_1$ *and* $R_2$.

(*b*) *The emission of a tungsten filament (current I per unit area) at absolute temperature T is given by the formula* $I = aT^2e^{-b/T}$ *where a, b are positive constants.*

(*i*) *Calculate the percentage increase in emission as the temperature rises from 2,400° to 2,500° absolute, assuming* $b = 5{\cdot}24 \times 10^4$ *and* $e = 2{\cdot}718$.

(*ii*) *Express b in terms of I, T and a.*

**A. 1.** (*a*) $$R_0 = \sqrt{R_1R_0 + \frac{R_1^2}{4}}.$$

(*b*) (*i*) Percentage increase $= \underline{159{\cdot}6.}$

(*ii*) $$b = -T \log_e \frac{I}{aT^2}.$$

**Q. 2.** (*a*) *Derive, without assuming the standard formula, the condition for the roots of the equation* $ax^2 + 2hx + b = 0$ *to be complex. Obtain also the conditions for the expression* $ax^2 + 2hxy + by^2$ *to be positive for all finite real values of x and y.*

(*b*) *Show that the equation* $x = c(x - 1)(x - 2)$ *has real roots only if c does not lie between* $-3$, $-2\sqrt{2}$ *and* $-3$, $+2\sqrt{2}$.

*Use this fact to sketch the graph* $y = \dfrac{x}{(x - 1)(x - 2)}$, *indicating any asymptotes which occur.*

(*Maximum and minimum values are not required.*)

**A. 2.** (*a*) For the roots to be complex $h^2 < ab$

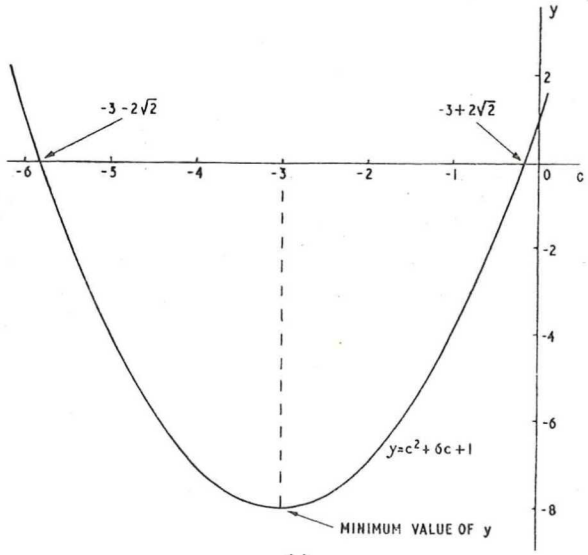
The simple conditions for the expression $ax^2 + 2hxy + by^2$ to be positive for all finite real values of $x$ and $y$ are that $a$ is positive and $ab \geqslant h^2$.

(*b*)
$$x = c(x-1)(x-2),$$
$$\text{or } x = cx^2 - 3cx + 2c,$$
$$\therefore cx^2 - x(3c+1) + 2c = 0,$$
$$\therefore x = \frac{3c + 1 \pm \sqrt{(3c+1)^2 - 8c^2}}{2c}.$$

The roots of the equation are real when
$$(3c+1)^2 \geqslant 8c^2$$
$$\text{i.e. } 9c^2 + 6c + 1 \geqslant 8c^2$$
$$\text{or } c^2 + 6c + 1 \geqslant 0$$

The roots of the equation $c^2 + 6c + 1 = 0$ are
$$c = \frac{-6 \pm \sqrt{36-4}}{2} = \frac{-6 \pm 4\sqrt{2}}{2}$$
$$= -3 + 2\sqrt{2} \text{ and } -3 - 2\sqrt{2}.$$

The function $y = c^2 + 6c + 1$ is parabolic and its graph is shown in sketch (*a*). As the coefficient of $c^2$ is positive, there is a minimum



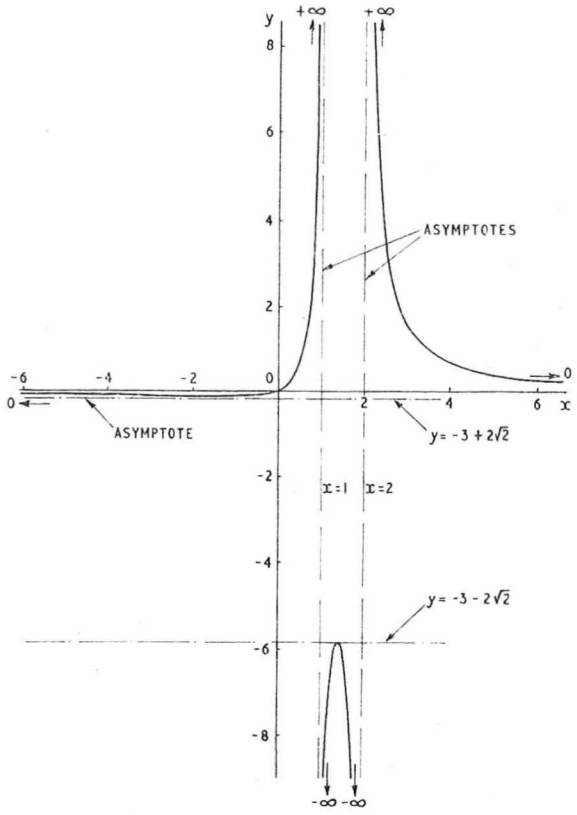

(*a*)

value at a value of $c$ mid-way between $-3 + 2\sqrt{2}$ and $-3 - 2\sqrt{2}$, i.e. when $c = -3$. Thus, between these values of $c$, $y$ will be negative. Hence $c^2 + 6c + 1$ can be positive, or equal to zero, only when $c$ does not lie between $-3 + 2\sqrt{2}$ and $-3 - 2\sqrt{2}$, and this is the condition for the roots of the equation $x = c(x-1)(x-2)$ to be real. Q.E.D.

$$y = \frac{x}{(x-1)(x-2)}.$$

Comparing this equation with $x = c(x-1)(x-2)$, the function $y$ may be equated to the function $c$. Thus $y$ is real only outside the range of values from $-3 - 2\sqrt{2}$ to $-3 + 2\sqrt{2}$, i.e. from $-5{\cdot}8284$ to $-0{\cdot}1716$. Hence, to sketch the graph of $y$, start by drawing the two lines $y = -3 - 2\sqrt{2}$ and $y = -3 + 2\sqrt{2}$, as shown in sketch (*b*), in the knowledge that no portion of the graph can occur in the area bounded by these lines.

From the factors forming the denominator, it may be seen that discontinuities will occur at $x = 1$ and $x = 2$ because for these values the denominator is zero and the value of $y$ becomes infinite. Just below $x = 1$ both factors of the denominator are negative and hence $y$ is positive. Thus, since $y = 0$ at $x = 0$, the curve will ascend as shown towards $+\infty$ at $x = 1$. As $x$ becomes negative, $y$ will always remain negative and will become increasingly small, because the denominator of $y$ increases faster than the numerator. At, say, $x = -1$, $y = -\frac{1}{6}$ and this, or a similar point, can be plotted to fix roughly the position of the curve for negative values of $x$. The curve is thus contained between the $x$-axis and the line $y = -3 + 2\sqrt{2}$. At $x \to -\infty$, $y \to 0$.




(*b*)

Similarly it may be deduced that, as $x$ reduces from just above 2 to the value 2, the curve rises again to $+\infty$. As $x \to \infty$, $y \to 0$ and hence the shape of the curve is as shown.

Over the range from $x = 1$ to $x = 2$, the value of $y$ will always be negative and will decrease towards $-\infty$ in the neighbourhood of $x = 1$ and, also, of $x = 2$.

A fact not asked for in the question is that $y$ will attain a maximum value of $-3 - 2\sqrt{2}$ when $x = \sqrt{2}$. It is not really possible to complete the sketch properly unless this is determined but it could be completed roughly on the assumption that a maximum must occur somewhere between $x = 1$ and $x = 2$.

The asymptotes are, as shown in sketch (*b*), $x = 1$, $x = 2$ and $y = 0$.

**Q. 3.** (*a*) *Assuming that the series* $1 - \frac{3}{4}x + \frac{3 \times 5}{4 \times 8}x^2 \ldots$ *is the expansion of the function* $(1 + kx)^n$, *find the values of the constants* $k$ *and* $n$, *and write down the next two terms of the series. For what range of values of* $x$ *is this expansion valid?*

(*b*) *In Fig.* 1 *AB, CD represent the heights* $h_1$ *and* $h_2$ *of a transmitting aerial and a receiving aerial respectively.*

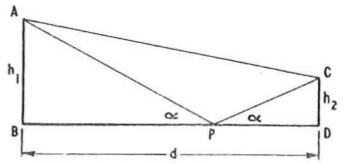



Fig. 1

*Ignoring the earth's curvature, show that the ground-reflected ray path,* $AP + PC$, *has a length* $l_1$ *given by* $l_1 = d\left[1 + \left(\frac{h_1 + h_2}{d}\right)^2\right]^{\frac{1}{2}}$

*Write down a similar expression for the length* $l_2$ *of the direct path* $AC$.

*If $h_1 + h_2$ is small compared with the distance d, show that the difference $l_1 - l_2$ is very nearly $\frac{2h_1h_2}{d}$.*

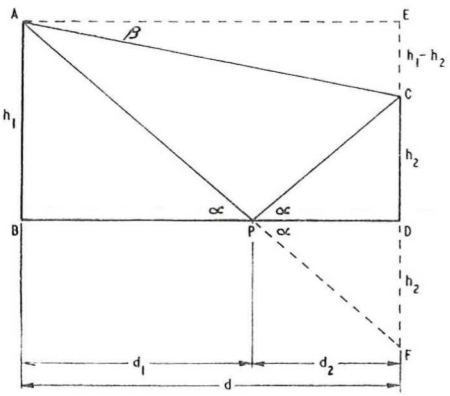
**A. 3.** (*a*) $k = \frac{1}{2}$ and $n = -\frac{3}{2}$.

The next two terms of the series are

$$-\frac{3 \times 5 \times 7}{2^6 \times 3!} \times x^3 \text{ and } \frac{3 \times 5 \times 7 \times 9}{2^8 \times 4!} \times x^4.$$

For the expansion to be valid, $|kx| < 1$, i.e. $\left|\frac{x}{2}\right| < 1$ or $|x| < 2$, where $|x|$ denotes the numerical value of $x$. Thus $x$ must be greater than $-2$ and less than $+2$.

(*b*) In the sketch, AP has been extended to cut CD produced at F and AE and CE, intersecting at E, are drawn parallel to BD and AB, respectively.




Now $\angle DPF = \angle APB = \alpha$ and $DF = CD = h_2$.

Also $EC = AB - CD = h_1 - h_2$.

Let $BP = d_1$ and $PD = d_2$.

Then $AP = \frac{d_1}{\cos\alpha}$ and $PC = \frac{d_2}{\cos\alpha}$.

$$\therefore l_1 = AP + PC = \frac{d_1 + d_2}{\cos\alpha} = d \sec\alpha.$$

Now $\angle EAP = \angle APB = \alpha$,

$$\therefore \tan\alpha = \frac{EF}{AE} = \frac{h_1 + h_2}{d}.$$

$$\therefore \sec\alpha = \sqrt{1 + \tan^2\alpha} = \left[1 + \left(\frac{h_1 + h_2}{d}\right)^2\right]^{\frac{1}{2}}$$

$$\text{and } l_1 = d\left[1 + \left(\frac{h_1 + h_2}{d}\right)^2\right]^{\frac{1}{2}}. \quad \text{Q.E.D.}$$

Similarly, from triangle AEC,

$$\tan\beta = \frac{EC}{AE} = \frac{h_1 - h_2}{d}$$

$$\therefore \sec\beta = \left[1 + \left(\frac{h_1 - h_2}{d}\right)^2\right]^{\frac{1}{2}}$$

$$\text{and } l_2 = AC = d \sec\beta$$

$$\text{or } l_2 = d\left[1 + \left(\frac{h_1 - h_2}{d}\right)^2\right]^{\frac{1}{2}},$$

which is similar to the expression for $l_1$.

$$\text{Now } l_1 - l_2 = d\left[1 + \left(\frac{h_1 + h_2}{d}\right)^2\right]^{\frac{1}{2}} - d\left[1 + \left(\frac{h_1 - h_2}{d}\right)^2\right]^{\frac{1}{2}}$$

$$\simeq d\left[1 + \frac{1}{2}\left(\frac{h_1 + h_2}{d}\right)^2 - \left\{1 + \frac{1}{2}\left(\frac{h_1 - h_2}{d}\right)^2\right\}\right],$$

expanding each of the binomial expressions to two terms only. These approximations will be true provided $h_1 + h_2$ is small compared with $d$, so that $\left(\frac{h_1 + h_2}{d}\right)^2$ and $\left(\frac{h_1 - h_2}{d}\right)^2$ will be quite small compared with 1.

$$\text{Thus } l_1 - l_2 \simeq d\left[1 - 1 + \frac{1}{2d^2}\left\{(h_1 + h_2)^2 - (h_1 - h_2)^2\right\}\right]$$

$$\simeq \frac{d}{2d^2}\left\{(h_1 + h_2 - h_1 + h_2)(h_1 + h_2 + h_1 - h_2)\right\},$$

$$= \frac{1}{2d}\left\{2h_2 \times 2h_1\right\},$$

$$= \frac{2h_1h_2}{d}. \quad \text{Q.E.D.}$$

*Note:* In question 3 (*b*), the index in the expression for $l_1$ should be $\frac{1}{2}$ and this was assumed in the answer. It is understood that, where the index appeared as 2, as it did on a number of papers, due allowance was made in the marking of papers from candidates who took 2 to be the correct index.

***Q.* 4.** *The following data was obtained in testing cement mixes having different values of the water/cement ratio:*

| *Water/cement ratio* $x$ .. | 0·4 | 0·5 | 0·6 | 0·7 | 0·8 |
|---|---|---|---|---|---|
| *Strength in tons/in².* $y$ .. | 2·24 | 1·77 | 1·31 | 1·01 | 0·76 |

*Show, by plotting suitable variables, that the formula $y = a\,e^{-kx}$ is a reasonable one for the range of x-values tested. Use the graph to obtain estimates of the constants a and k.*

*Could the constant a have any physical significance?*

***Q.* 5.** *The cost of erecting a new office block is estimated at £10,000 for the ground floor, £10,500 for the first floor, £11,000 for the second floor, and so on. In addition, other overheads (assumed constant) are £70,000. Find the cost of erecting a building with n floors.*

*If the annual income from each floor is £1,000, how many floors would give the greatest percentage income on the outlay ?*

**A. 5.** Total cost = £70,000 + £10,000 + £10,500 + £11,000 + . . .

Disregarding the first term of £70,000, the remaining terms form an arithmetic progression with a common difference of £500. The first term, $a$, of this progression is £10,000 and the sum of $n$ terms is given by $\frac{n}{2}\left\{2a + (n-1)d\right\}$ where $d$ = £500 is the common difference.

$$\text{Hence, total cost} = £70{,}000 + £\frac{n}{2}\left\{20{,}000 + (n-1)\,500\right\}$$

$$= £\left(70{,}000 + 10{,}000n + 250n^2 - 250n\right)$$

$$= £\left(70{,}000 + 9{,}750n + 250n^2\right)$$

$$= £\,250\left\{280 + 39n + n^2\right\}.$$

The income from $n$ floors will be £1,000$n$.

The income referred as a percentage of the outlay

$$= \frac{1{,}000n}{250\,(280 + 39n + n^2)} \times 100 \text{ per cent}$$

$$= \frac{400n}{280 + 39n + n^2} \text{ per cent.}$$

Let this percentage income be $I$.

$$\text{Then } \frac{dI}{dn} = \frac{(280 + 39n + n^2)\,400 - 400n\,(39 + 2n)}{(280 + 39n + n^2)^2}.$$

For $I$ to be a maximum or a minimum, $\frac{dI}{dn} = 0$.

$$\therefore (280 + 39n + n^2)\,400 - 400n\,(39 + 2n) = 0$$
$$\text{or } 280 + 39n + n^2 - 39n - 2n^2 = 0.$$
$$\therefore n^2 = 280$$
$$\text{or } n = \sqrt{280} = 16{\cdot}73.$$

To test whether $n = 16{\cdot}73$ gives a maximum or a minimum value, substitute values of $n$ close to 16·73 in the expression for $I$.

Thus, when $n = 16$,

$$I_{16} = \frac{400 \times 16}{280 + 39 \times 16 + 16^2}$$
$$= \frac{6{,}400}{280 + 624 + 256}$$
$$= \frac{6{,}400}{1{,}160} = 5{\cdot}517.$$

When $n = 15$,

$$I_{15} = \frac{400 \times 15}{280 + 39 \times 15 + 15^2}$$
$$= \frac{6{,}000}{280 + 585 + 225}$$
$$= \frac{6{,}000}{1{,}090} = 5{\cdot}505.$$

Thus $I_{15}$ is slightly less than $I_{16}$, from which it is seen that a maximum value of $I$ occurs when $n = 16{\cdot}73$.

The nearest integral number to 16·73 is 17 and hence the practical number of floors to give the greatest percentage income is 17.

*Q.* 6. (*a*) *Expand* $\sin 3A$ *in terms of powers of* $\sin A$.

(*b*) *Prove that* $\frac{\sin 3\theta}{1 + 2\cos 2\theta} = \sin\theta$. *Hence, or otherwise, show that* $\sin 15° = \frac{\sqrt{3} - 1}{2\sqrt{2}}$.

(*c*) *Calculate the values of* $\phi$ *between* $-180°$ *and* $+180°$ *which satisfy the equation* $5\cos\phi = 2 + \sin\phi$.

A. 6. (*a*) $\sin 3A = 3\sin A - 4\sin^3 A$.

(*b*) $$\frac{\sin 3\theta}{1 + 2\cos 2\theta} = \sin\theta.$$

$$\text{Left-hand side} = \frac{3\sin\theta - 4\sin^3\theta}{1 + 2(1 - 2\sin^2\theta)}$$

from the expansions used in part (*a*).

$$\text{Left-hand side} = \frac{\sin\theta\,(3 - 4\sin^2\theta)}{3 - 4\sin^2\theta},$$
$$= \sin\theta. \qquad \text{Q.E.D.}$$

Using the identity proved above,

$$\sin 15° = \frac{\sin 45°}{1 + 2\cos 30°},$$

$$\text{but } \sin 45° = \frac{1}{\sqrt{2}} \text{ and } \cos 30° = \frac{\sqrt{3}}{2}.$$

$$\therefore \sin 15° = \frac{\frac{1}{\sqrt{2}}}{1 + 2 \times \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{2}\,(1 + \sqrt{3})}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{2}\,(\sqrt{3} + 1)\,(\sqrt{3} - 1)} = \frac{\sqrt{3} - 1}{2\sqrt{2}}. \qquad \text{Q.E.D.}$$

(*c*) $$5\cos\phi = 2 + \sin\theta.$$
$$\text{or } 1\sin\phi - 5\cos\phi = -2$$

$$\therefore \sqrt{1^2 + 5^2}\left\{\frac{1}{\sqrt{26}}\sin\phi - \frac{5}{\sqrt{26}}\cos\phi\right\} = -2.$$

Let $\alpha = \tan^{-1}\frac{5}{1}$ so that $\sin\alpha = \frac{5}{\sqrt{26}}$ and $\cos\alpha = \frac{1}{\sqrt{26}}$.

Then $\sin\phi\cos\alpha - \cos\phi\sin\alpha = \frac{-2}{\sqrt{26}}$,

$$\text{or } \sin(\phi - \alpha) = \frac{-2}{5{\cdot}099} = -0{\cdot}3922.$$

$$\therefore \phi - \alpha = -23°6' \text{ or } -156°54',$$

$$\text{but } \alpha = \tan^{-1}5 = \sin^{-1}\frac{5}{5{\cdot}099} = 78°42'.$$

$$\therefore \phi = -23°6' + 78°42' \text{ or } -156°54' + 78°\,42'$$
$$= 55°36' \text{ or } -78°12'.$$

These are the only values of $\phi$ between $-180°$ and $+180°$ which satisfy the equation.

*Q.* 7. (*a*) *If* $y = \cot 2x$, *derive from first principles an expression for* $dy/dx$.

(*b*) *Differentiate with respect to* $x$ *the functions*

(*i*) $\log_e(\operatorname{cosec} x + \cot x)$,

(*ii*) $\frac{x^3}{\sqrt{1 - x^2}}$.

(*c*) *If* $x = e^{-3t}\cos 4t$, *derive an expression for* $d^2x/dt^2$ *in the form* $pe^{-3t}\cos(4t - \beta)$, *where* $p$ *is a constant and* $\cot\frac{1}{2}\beta = 0{\cdot}75$.

A. 7. (*a*) $y = \cot 2x$.

Let $x$ increase by a small amount $\delta x$ and let $\delta y$ be the corresponding change in $y$.

$$\text{Then } y + \delta y = \cot 2(x + \delta x)$$
$$\text{or } \delta y = \cot 2(x + \delta x) - \cot 2x, \text{ since } y = \cot 2x.$$
$$= \frac{\cos 2(x + \delta x)}{\sin 2(x + \delta x)} - \frac{\cos 2x}{\sin 2x},$$
$$= \frac{\sin 2x \times \cos 2(x + \delta x) - \cos 2x \times \sin 2(x + \delta x)}{\sin 2x \times \sin 2(x + \delta x)},$$
$$= \frac{\sin\{2x - 2(x + \delta x)\}}{\sin 2x \times \sin 2(x + \delta x)},$$
$$= \frac{\sin(-2\delta x)}{\sin 2x \times \sin 2(x + \delta x)}.$$
$$\therefore \frac{\delta y}{\delta x} = \frac{\frac{\sin(-2\delta x)}{\delta x}}{\sin 2x \times \sin 2(x + \delta x)},$$
$$= \frac{-2 \times \frac{\sin(-2x)\delta}{-2\delta x}}{\sin 2x \times \sin 2(x + \delta x)}$$
$$\therefore \frac{dy}{dx} = \lim_{\delta x \to 0}\frac{\delta y}{\delta x}.$$
$$= \frac{-2}{\sin^2 2x}, \text{ since } \lim_{\theta \to 0}\frac{\sin\theta}{\theta} = 1,$$
$$= -2\operatorname{cosec}^2 2x.$$

(*b*) (*i*) $$\frac{d\left\{\log_e(\operatorname{cosec} x + \cot x)\right\}}{dx}$$
$$= \frac{1}{\operatorname{cosec} x + \cot x} \times (-\operatorname{cosec} x\cot x - \operatorname{cosec}^2 x),$$

$$= \frac{-\operatorname{cosec} x\,(\cot x + \operatorname{cosec} x)}{\operatorname{cosec} x + \cot x},$$

$$= \underline{-\operatorname{cosec} x.}$$

(*ii*) $$\frac{d}{dx}\left\{\frac{x^3}{\sqrt{1-x^2}}\right\} = \frac{(1-x^2)^{\frac{1}{2}} \times 3x^2 - x^3 \times \frac{d(1-x^2)^{\frac{1}{2}}}{dx}}{1-x^2},$$

$$= \frac{(1-x^2)^{\frac{1}{2}} \times 3x^2 - x^3 \times \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \times (-2x)}{1-x^2}$$

$$= \frac{(1-x^2) \times 3x^2 + x^4}{(1-x^2)^{\frac{3}{2}}},$$

$$= \frac{3x^2 - 2x^4}{(1-x^2)^{\frac{3}{2}}} = \underline{\frac{x^2(3-2x^2)}{(1-x^2)^{\frac{3}{2}}}}.$$

(*c*) $$x = e^{-3t}\cos 4t.$$

$$\frac{dx}{dt} = e^{-3t}(-4\sin 4t) - 3e^{-3t}\cos 4t$$

$$= -e^{-3t}\left\{3\cos 4t + 4\sin 4t\right\}$$

$$= -5e^{-3t}\left\{\frac{3}{5}\cos 4t + \frac{4}{5}\sin 4t\right\}$$

$$= -5e^{-3t} \times \cos(4t - \alpha),$$

where $\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$ and hence $\tan\alpha = \frac{4}{3}$.

$$\frac{d^2x}{dt^2} = 15e^{-3t}\cos(4t-\alpha) - 5e^{-3t}(-4)\sin(4t-\alpha)$$

$$= 5e^{-3t}\left\{3\cos(4t-\alpha) + 4\sin(4t-\alpha)\right\}.$$

This is of the same form as the expression for $\frac{dx}{dt}$ and, therefore

$\frac{d^2x}{dt^2} = 25e^{-3t}\cos(4t-2\alpha)$, where $\alpha = \cos^{-1}\frac{3}{5}$ as before.

$\therefore \frac{d^2x}{dt^2}$ is expressed in the form $pe^{-3t}\cos(4t-\beta)$, where $p = 25$ and $\alpha = \frac{\beta}{2}$ when $\cot\frac{\beta}{2} = \frac{1}{\tan\alpha} = 0{\cdot}75$. Q.E.D.

**Q. 8.** (*a*) *Calculate* (*i*) *the mean value,* (*ii*) *the root-mean-square value of the function* $40(1 - e^{-pt})$ *over the interval* $t = 0$ *to* $t = \frac{0}{p}$.

(*b*) *Sketch the waveform represented by the equation*

$$y = 6\cos\omega t - 2\cos\omega t \text{ from } t = 0 \text{ to } t = \frac{2\pi}{\omega} \text{ seconds.}$$

*Find by calculation when* $y$ *is first a maximum within this range, if* $\omega = 10^4$ *radians per second. What is this maximum value?*

*Confirm this by use of the second differential.*

**A. 8.** (*b*) $y = 6\cos\omega t - 2\cos\omega t$

*Note:* As printed in the question paper, $y = 4\cos\omega t$ and the waveform is that of a normal sine function, with a maximum value of 4 occurring first at $t = 0$.

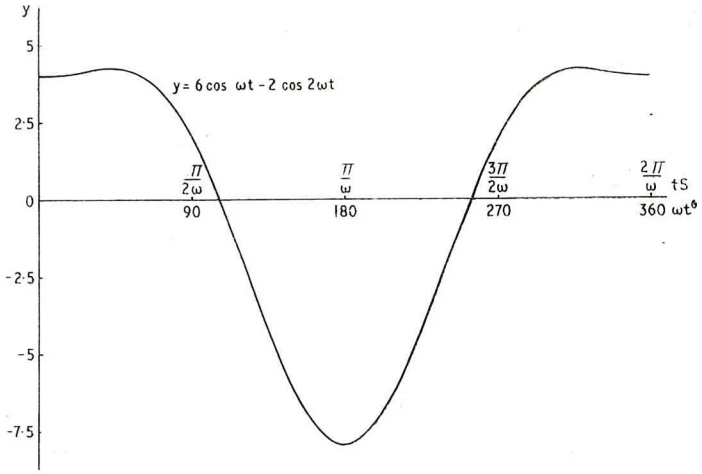
Judging by similar questions set in previous papers, however, it seems almost certain that there was a typographical error in the question and that the function $y$ was possibly intended to be $6\cos\omega t - 2\cos 2\omega t$. The following answer is based upon this assumption.

$$y = 6\cos\omega t - 2\cos 2\omega t.$$

As $t$ varies from 0 to $\frac{2\pi}{\omega}$ seconds, $\omega t$ varies from 0 to $2\pi$ radians, i.e. from 0° to 360°. The waveform may be sketched from the following table of values.

| $\omega t°$ | 0 | 30 | 60 | 90 | 120 | 150 | 180 |
|---|---|---|---|---|---|---|---|
| $\cos\omega t$ | 1 | 0·866 | 0·5 | 0 | − 0·5 | − 0·866 | − 1 |
| $6\cos\omega t$ | 6 | 5·196 | 3 | 0 | − 3 | − 5·196 | − 6 |
| $\cos 2\omega t$ | 1 | 0·5 | − 0·5 | − 1 | − 0·5 | 0·5 | 1 |
| $2\cos 2\omega t$ | 2 | 1 | − 1 | − 2 | − 1 | 1 | 2 |
| $y$ | 4 | 4·196 | 4 | 2 | − 2 | − 6·196 | − 8 |

The table only covers the range of $\omega t$ from 0° to 180°. As $\cos\theta = \cos(-\theta)$, the range of $y$-values from 180° to 360° is the same as that from 0° to 180°, when written down in reverse order, e.g. $y_{210°} = y_{150°}$, $y_{240°} = y_{120°}$ and so on.




The waveform is shown in the sketch.

$$\frac{dy}{dt} = -6\omega\sin\omega t + 4\omega\sin 2\omega t.$$

For maximum or minimum values, $\frac{dy}{dt} = 0$.

$$\therefore 6\omega\sin\omega t = 4\omega\sin 2\omega t,$$

$$\text{or } 3\sin\omega t = 4\sin\omega t\cos\omega t.$$

$$\therefore \sin\omega t = 0 \text{ when } \omega t = 0° \text{ or } 180°,$$

$$\text{or } \cos\omega t = \frac{3}{4} \text{ when } \omega t = \pm 41°24'.$$

From the sketch, the first maximum occurs when $\omega t = 41°24'$.

$$\therefore \omega t = 0{\cdot}7226 \text{ radians},$$

$$\text{or } t = \frac{0{\cdot}7226}{10^4} \text{ seconds}$$

$$= 72{\cdot}26\,\mu\text{s}.$$

Thus the first maximum occurs after $\underline{72{\cdot}26\,\mu\text{s}}$.

$$y_{max} = 6\cos 41°24' - 2\cos 82°48'$$

$$= 6 \times \frac{3}{4} - 2 \times 0{\cdot}1253$$

$$= \underline{4{\cdot}249.}$$

$$\frac{d^2y}{dt^2} = -6\omega^2\cos\omega t + 8\omega^2\cos 2\omega t$$

When $\omega t = 41°24'$,

$$\frac{d^2y}{dt^2} = -6\omega^2 \times \frac{3}{4} + 8\omega^2 \times 0{\cdot}1253$$

$$= \omega^2(-4{\cdot}5 + 1{\cdot}0024)$$

$$\simeq -3{\cdot}5\,\omega^2.$$

As the value of the second derivative is negative, this confirms that the corresponding value of $y$ is a maximum.

**Q. 9.** (*a*) *Evaluate:*

$$(i)\ \int_4^5 \frac{dx}{(2x-7)^2} \qquad (ii)\ \int_0^{\frac{\pi}{4}} \cos 3x \cos x\, dx$$

(*b*) *If* $y = \log_e[x + \sqrt{(1+x^2)}]$, *prove that* $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

**A. 9.** (*a*) (*i*) $\int_4^5 \frac{dx}{(2x-7)^2} = \int_4^5 (2x-7)^{-2}\,dx = \underline{\frac{1}{3}}$.

(*ii*) $\int_0^{\frac{\pi}{4}} \cos 3x \cos x\,dx = \underline{\frac{1}{4}}$.

(*b*) $y = \log_e\left[x + (1+x^2)^{\frac{1}{2}}\right]$.

$$\frac{dy}{dx} = \frac{1}{x+(1+x^2)^{\frac{1}{2}}} \times \left\{1 + \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \times 2x\right\},$$

$$= \frac{x+(1+x^2)^{\frac{1}{2}}}{(1+x^2)^{\frac{1}{2}}\left\{x+(1+x^2)^{\frac{1}{2}}\right\}} = (1+x^2)^{-\frac{1}{2}}.$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}(1+x^2)^{-\frac{3}{2}} \times 2x = \frac{-x}{(1+x^2)^{\frac{3}{2}}}$$

Hence,

$$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{-(1+x^2)x}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{(1+x^2)^{\frac{1}{2}}}$$

$$= \frac{-x}{(1+x^2)^{\frac{1}{2}}} + \frac{x}{(1+x^2)^{\frac{1}{2}}} = 0. \qquad \text{Q.E.D.}$$

**Q. 10.** (*a*) *If* $z_1 = r_1 \underline{/\theta_1}$ *and* $z_2 = r_2 \underline{/\theta_2}$ *are two complex numbers given in polar form, state and prove a formula expressing the quotient* $\frac{z_1}{z_2}$ *in polar form.*

(*b*) *Express* $z_1 = \frac{1-j\sqrt{3}}{1+j}$ *in the form* $a + jb$ *and show that* $ab$ *is rational.*

(*c*) *Find also the argument and modulus of* $z_1$.

**A. 10.** (*a*) $z_1 = r_1 \underline{/\theta_1}$ and $z_2 = r_2 \underline{/\theta_2}$

$$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}\right)\underline{/\theta_1 - \theta_2}. \qquad \text{Q.E.D.}$$

This is proved as follows:
Expressing the two numbers in the full polar form,

$$z_1 = r_1(\cos\theta_1 + j\sin\theta_1),$$

and $z_2 = r_2(\cos\theta_2 + j\sin\theta_2)$.

$$\therefore \frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + j\sin\theta_1)}{r_2(\cos\theta_2 + j\sin\theta_2)}$$

$$= \frac{r_1(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)}{r_2(\cos\theta_2 + j\sin\theta_2)(\cos\theta_2 - j\sin\theta_2)}$$

$$= \frac{r_1\left\{\begin{array}{l}(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) \\ \quad + j(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)\end{array}\right\}}{r_2(\cos^2\theta_2 + \sin^2\theta_2)}$$

$$= \frac{r_1}{r_2}\left\{\cos(\theta_1-\theta_2) + j\sin(\theta_1-\theta_2)\right\}$$

$$= \underline{\left(\frac{r_1}{r_2}\right)/\theta_1 - \theta_2.}$$

(*b*) $z_1 = \frac{1-j\sqrt{3}}{1+j}$,

$$= \frac{(1-j\sqrt{3})(1-j)}{(1+j)(1-j)}$$

$$= \frac{1-\sqrt{3}-j\sqrt{3}-j}{1+1}$$

$$= \underline{\frac{1-\sqrt{3}}{2} - j\frac{1+\sqrt{3}}{2}.}$$

This is of the form $a + jb$.

$$ab = \frac{1-\sqrt{3}}{2} \times \frac{-(1+\sqrt{3})}{2},$$

since $a = \frac{1-\sqrt{3}}{2}$ and $b = -\frac{(1+\sqrt{3})}{2}$.

$$\therefore ab = \frac{-\left\{1-(\sqrt{3})^2\right\}}{4}$$

$$= \frac{-\{1-3\}}{4} = \frac{1}{2} \text{ which is rational.}$$

(*c*) The modulus of $z_1 = \sqrt{\left(\frac{1-\sqrt{3}}{2}\right)^2 + \left(\frac{1+\sqrt{3}}{2}\right)^2}$

$$= \frac{1}{2}\sqrt{1-2\sqrt{3}+3+1+2\sqrt{3}+3}$$

$$= \frac{1}{2}\sqrt{8}$$

$$= \underline{\sqrt{2}}.$$

If $\theta_1$ is the argument of $z_1$,

$$\tan\theta_1 = \frac{-\frac{(1+\sqrt{3})}{2}}{\frac{1-\sqrt{3}}{2}} = \frac{-(1+\sqrt{3})}{1-\sqrt{3}},$$

$$= -\frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = -\frac{1+2\sqrt{3}+3}{1-3},$$

$$= 2+\sqrt{3} = 3{\cdot}732.$$

$$\therefore \theta_1 = 75° \text{ or } (180° + 75°).$$

Since $a$ and $b$ are both negative, $\theta_1$ must lie in the third quadrant and hence:

Argument of $z_1 = \underline{255°}$.

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## LINE PLANT PRACTICE B, 1964

**Students were expected to attempt not more than any six questions.**

**Q. 1.** *Describe the construction of a loading pot of a type that would be used for loading a 542-pair 20-lb P.C.Q.T. cable in a manhole. What protective measures are taken to prevent damage to the stub cables whilst in transit?*

**A. 1.** A loading pot suitable for loading a 542-pair, 20 lb/mile P.C.Q.T. cable and for installation in a manhole consists of a welded rectangular case constructed of $\frac{3}{16}$ in. sheet steel. The dimensions of the case are such as to permit its passage, without damage, through a standard manhole frame opening. Two $\frac{3}{4}$ in. mild-steel channels are fitted to the base of the case to allow it to stand with the base clear of the manhole floor. The lid of the case is provided with two entry glands for the stub cables, a cable-supporting bracket, at least two lifting rings or eye-bolts, and one or two holes to enable the case to be filled with compound. Before any coils are inserted, the case (including lid) is proved free of holes with an air pressure of not less than 20 lb/in². The 542 coils, of appropriate grades, are assembled in the case and the windings are connected to the conductors of the stub cables which enter the case

via, and are plumbed to, the two entry glands on the lid. The cables are clamped firmly, but without deformation, by the supporting brackets. The two wires from one end of the windings of each coil are connected to a separate pair in one stub cable and the two wires from the other end of the windings are connected to the corresponding pair in the other stub cable, each winding being connected to the same conductor of the corresponding pairs in both stub cables. The stub cables, one of 8 ft, the other of 6 ft, are each sheathed with tin/antimony lead (alloy "E") of 0·15 in. minimum thickness and contain 544, 6½ lb copper-conductor pairs arranged in quad and layer formations. Identification of the pairs is provided by the conventional quad colour scheme and, when looking at the free ends of the stub cables, the colour rotations are anti-clockwise. Each conductor is lapped with two paper tapes applied in opposite directions. Spare pairs in one stub cable are directly connected, within the case, to the corresponding spare pairs in the other stub cable. A special impregnating compound is run into the case, via the filling holes, under pressure or by a vacuum process so as to completely cover the coils, wiring, and cable ends within the case and to render them impervious to moisture. Before topping up with the compound, a pressure test of 20 lb/in². may be carried out on the completed assembly, making use of the filling holes; alternatively this test may be carried out before the compound is run into the case. After topping up, the filling holes are suitably sealed. To resist corrosion, the outside of the case and external ironwork are thoroughly cleaned by shot blasting and, within one hour of this treatment, are then zinc sprayed to provide a protective coating of at least 5 mils thick. External fittings are painted with black paint. In addition, the case is given a coating of bitumen compound, embedded in which is a tight-fitting, strong, open-weave hessian bag to strengthen the coating. Care is taken to ensure that the hessian is completely covered and that none of the fibres protrude. A brass label is affixed to the lid of the case or to the cable-supporting brackets and indicates the manufacturer's name, serial number of the case, year of manufacture, number of coils, their inductance and grade and the stub-cable quads not connected to coils. The manufacturer, at his discretion, may provide 2 additional coils, connected to spare quads, in the assembly to make up for a faulty quad (including faulty associated coils) which may be found during electrical tests of the assembly. Details of additional coils are given on the brass label and if a faulty quad exists the manufacturer gives details on a lead collet fitted to one of the stub cables between the gland-wipe and the cable-support.

As a protective measure against damage while the loading pot is in transit the stub cables are coiled on a former (usually metal and attached to the cable brackets) and tied securely in position. The free ends of the stub cables are plumbed to seal them against the ingress of moisture. The diameter of the coil is 24 in. and a wrapping of hessian tape is provided. In addition, during transportation by rail and sometimes by road, the complete loading-pot assembly is contained in a packing case. Except for the packing case, which should be returned without delay after receipt, any protection which is provided for both the pot and stub cables should be retained in position during storage and for as long as is practicable during the installation operations.

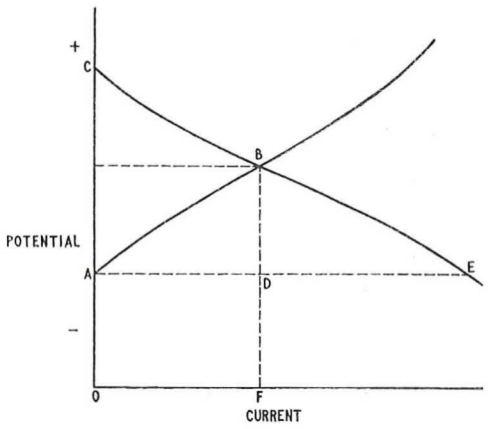
*Q.* 2. *The pairs in a paper-insulated junction cable have a normal insulation resistance of* 1,000 *megohms between each wire and earth. Due to an incipient fault on the cable however, the insulation of a few of the wires has fallen, and in the worst pair measures* 100 *megohms. Describe in detail how such a fault could be located.*

*Include in your answer the electrical values of the components of the measuring apparatus used.*

*Q.* 3. *Explain how electrolytic corrosion of lead-sheathed cables can sometimes be prevented by changing the electrical potential of the sheath. Describe an installation for achieving this object by the application of current from a central source.*

A. 3. The surface of a lead cable sheath is never perfectly uniform, nor is the duct water in which the cable may be immersed. The intrinsic grain structure of lead, breaks in oxide films, differential aeration, variations in the electrolyte, dissimilar-metal contacts, etc. all contribute to the formation of anodic and cathodic areas on the surface of the cable sheath. Minute electric currents circulate from anodic areas, through the electrolyte, to cathodic areas; electrolytic corrosion and pitting occurring at the anodic areas. The polarization curves of an anode and cathode are shown in the sketch. Points A and C represent the open-circuit potentials of the anode and cathode, respectively, whilst B is the mean potential of the lead sheath when a current OF is flowing between anode and cathode.

If a current is passed on to the lead sheath from an external source, the potential of the sheath is lowered from B towards D and the anode is depolarized along curve BA, whilst the cathode is polarized further along curve BE. When potential D is reached the current leaving the anode becomes zero and corrosion ceases. A further decrease in potential will reverse the direction of current at the anode and so make it a cathode.




This principle is known as cathodic protection and is carried out in practice by connecting the negative terminal of a mains transformer-rectifier to the cable sheath to be protected and the positive terminal to a sacrificial anode or groundbed. The rectifier may be housed in a telephone exchange, repeater station or a roadside cabinet or it may be pole-mounted; in the last two situations a special electricity service will be needed. The current output of the rectifier may be 2, 5, 10 or 15 amp depending upon the extent of plant to be protected but the voltage should not normally exceed 50 volts. The output is not smoothed unless noise interference on telephone circuits is found.

The design and siting of the groundbed are factors contributing in no small way to the success of a cathodic protection scheme. Groundbeds of a decade ago consisted of scrap steel rails surrounded by coke back-fill laid in trenches in star formation with a jointing box at the centre, the coke serving the dual purpose of reducing the corrosion rate of the steel core and of reducing the resistance to earth of the groundbed. Graphite rods were also tried but were not wholly successful. Silicon-iron rods were a vast improvement and are still widely used and in recent years platinized titanium in the form of wires, rods and meshes has come into prominence. With proper design and handling, the film of platinum appears to be virtually indestructible. A modern groundbed may take the following forms:

(*a*) a long trench with 12 in. of coke in the bottom with platinized titanium mesh anodes inserted at intervals of 10 to 15 ft, or

(*b*) a 4 in. or 6 in. diameter borehole about 20 ft deep in which is inserted a platinized titanium wire surrounded by coke dust.

To reduce the risk of failure the connexions between the anodes and the connecting cable are enclosed in a moulding of epoxy resin.

The groundbed should be sited as far as is practicable from the cables to be protected and from other underground metallic plant, the site chosen being in ground of low soil resistivity and permanently damp if possible.

To ensure that the cable sheaths are adequately protected, potential tests are made at a number of points on the cable network by measuring the voltage, on a high-resistance voltmeter, between the lead sheath and a copper–copper sulphate reference electrode pushed into the soil close to the cable. Protection is achieved if the cable is made somewhat more negative than −620 mV to the reference electrode.

*Q.* 4. *Distinguish between the effects of fundamental-frequency and harmonic-frequency induction from power circuits into communication circuits. For each type of induction give a brief description of typical circumstances which could give rise to the interference, and briefly explain how its effects could be mitigated.*

A. 4. Fundamental-frequency induction from a power circuit into a communication circuit takes place when an earth fault occurs on the power circuit and where there is parallelism between the power circuit and the communication circuit. In Great Britain it is the practice for the neutral point of a high-voltage power system to be earthed at the transformer station which feeds the system. When the system is operating normally, the instantaneous value of the current in any conductor is equal, but flowing in the opposite direction, to the sum of the currents in the other conductors, and induction effects from the power conductors therefore tend to cancel. If, however, an earth fault

takes place on a conductor a current will flow through the earth back to the transformer station and the balance of the currents will be upset. As a result, a magnetic field will be produced along the route of the power line from the location point of the fault back to the transformer station and any communication lines having parallelism with the power line will have a voltage induced along them.

The magnitude of the voltage $E$ induced along a communication line is given by the formula

$$E = 2\pi fIM,$$

where $I$ is the earth fault current in amperes, $M$ the total mutual inductance between the two lines in henrys and $f$ the fundamental frequency of the power system (50 c/s in Great Britain). The value of $M$ decreases with an increase in separation between the two lines.

With close parallelism and heavy fault currents, induced voltages may be high enough to give rise to the risk of electric or acoustic shock to telephone users or staff and damage to cables and apparatus. To reduce the danger it is necessary to restrict the induced voltages to acceptable limits. One method is for the power authority to limit the fault current which can flow through the earth. This can be effected by the insertion of a resistor or inductor in the neutral-point connexion to earth at the supply-transformer station.

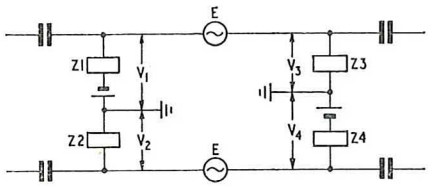
Alternatively, the communication line can be protected against the occurrence of high induced voltages as follows:

(*a*) gas discharge tubes can be fitted to the communication circuits at both ends and at selected points throughout the exposed length. These break down and earth the circuits when an induced voltage appears.

(*b*) Sectionalizing transformers can be fitted in each circuit at points throughout the exposed length, the spacing being such that the voltage between transformers is not excessive.

Harmonic-frequency induction from a power circuit can cause audible interference with a communication circuit. Again there must be parallelism between the two sets of plant. Odd harmonics are generally involved because even harmonics do not appear in normal a.c. systems.

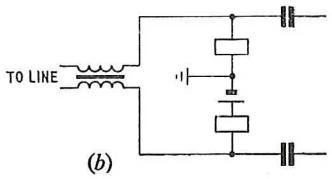
In a 3-phase power line third-harmonic currents and currents which are odd multiples of the third harmonic are in phase in the three line conductors. The three conductors may thus be regarded as being in parallel with a path to earth, which is either via the capacitances to earth of the line conductors, or directly to earth if the neutral point at the "receiving end" of the power line is earthed. In each wire of a parallel telephone line the earth-return harmonic current induces a longitudinal voltage, $E$, the magnitude of which depends on the value of the current and the separating distance between the wire and the power line. In the two wires of a telephone pair the induced voltages will be $E$ and $(E + \delta E)$, the difference, $\delta E$, being due to the different separations of the two wires from the power line. If the telephone circuit is perfectly balanced the voltage $\delta E$ will act round the loop and give rise to noise. The value of $\delta E$ may be reduced, however, by transposition or twisting of the pair, and in practice $\delta E$ is negligible. The major cause of noise is then the independent effects of equal voltages $E$ in the two legs of the circuit when it is unbalanced to earth; this usually occurs in the terminations. This condition is shown in sketch (*a*),




(*a*)

where the line wires are well balanced and have uniform electrical characteristics. It will be appreciated that if $Z_1 = Z_2$ and $Z_3 = Z_4$, then $V_1 = V_2$ and $V_3 = V_4$ and there is therefore no transverse voltage appearing as noise across the circuit. If, however, this impedance balance to earth does not exist then a noise voltage will appear across the circuit.

A simple method of reducing transverse voltages, and thus noise, due to unbalance in the termination is to insert a longitudinal inductor in the circuit, as shown in sketch (*b*). This inductor consists essentially of two balanced windings on a common core, connected so as to be inductive to longitudinal currents and non-inductive to loop currents. The inductor, therefore, offers a high impedance to longitudinal currents, reducing their values, and thus reducing the value of the noise voltage appearing across the termination.




(*b*)

Induction into communication circuits may also arise from:

(*a*) the presence of large harmonic phase currents, the external magnetic fields of which do not entirely cancel due to the spacing of the power conductors, and

(*b*) the contact-wire currents of a.c. electrified railways, which are particularly rich in harmonics.

***Q.* 5.** *Explain what is meant by the term "Annual Charge", which is used in connexion with the method of making economic comparisons by means of present values of annual charges. State the factors which make up the annual charge of a particular item, and explain in particular the effect of "Residual Value".*

**A. 5.** The inclusive cost of provision, maintenance and recovery, at the end of its average life, of an item of plant can be expressed in terms of an equal sum of money paid at the end of each year for the average life of the plant. This annual sum is known as an annual charge and is in respect of interest, depreciation, maintenance and, where applicable, operating costs.

The present value of the annual charge is the amount of money which, if invested now at the long-term interest rate, would enable the annual charge to be paid at the end of each year over a given period of time. This period of time is known as the costing period. For external plant, the annual charge is derived from annual-charge rates, which are expressed as percentages of the inclusive capital cost of the plant.

The interest rate is the long-term average interest rate and is the average of the interest paid on capital raised over a long period.

The maintenance rate is the average annual rate of maintaining the plant in serviceable condition over its average life.

The depreciation rate is the rate at which money is required to be invested annually in an interest-bearing sinking fund, so that at the end of the average life of the plant an amount, together with the net residual value of the plant, is available either to renew the plant or to pay back the original borrowed capital.

The residual value of an item of plant is the average value of the plant on recovery at the end of its average life in one position and is expressed as a percentage of the inclusive capital cost. It is the value of the recovered stores, less the labour cost of recovering, together with indirect and overhead charges. The value of the recovered stores includes a proportion of the items regarded as fit for re-issue.

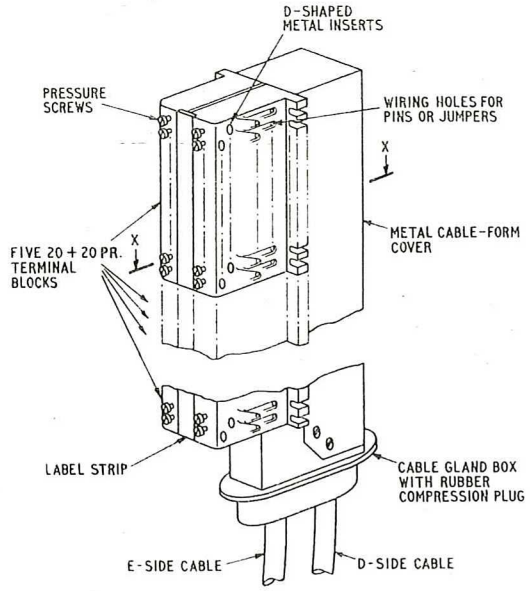
When the credit for the recovered stores, e.g. lead cable, is greater than the labour cost of recovery, the residual value is positive and the amount to be accrued in the sinking fund is reduced, so the annual charge for depreciation is reduced. Conversely, when the labour cost of recovery is greater than the value of the recovered stores, e.g. overhead wire, then the residual value is negative and the amount to be accrued in the sinking fund has to be increased; thus the depreciation annual charge is increased.

***Q.* 6.** *Describe the construction of an assembly used in a cabinet or pillar and give details of the terminal block used. What is the function of such an assembly?*

**A. 6.** A typical assembly is illustrated in sketch (*a*). It consists essentially of five terminal blocks mounted vertically in a metal cable-form cover with a cable gland box at the lower end. The terminated cable tails are supplied with the unit in the form of a continuous 30-ft or 60-ft loop which may be cut at a convenient point when the assembly is jointed into the cable network. The cable gland box has a rubber compression plug, which provides the mechanical clamping of the cables and for sealing against the ingress of moisture when the assembly is used in a pillar in conjunction with a special mounting. A simpler form of mounting with jumper rings is screwed to the back of the assembly when used in a cabinet.

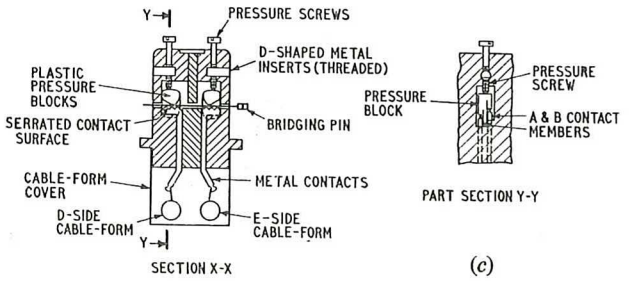
The terminal blocks are designed to provide a permanent termination for the two 100-pair/4-lb/mile polythene-insulated and sheathed cables by means of soldered connexions at the rear of the blocks under the cable-form cover. The blocks also provide a convenient mechanical means of random interconnexion between the pairs of the two cables, using jumper wires or bridging pins as described below.

Each terminal block comprises two main plastic mouldings mounted side by side, each being the mirror image of the other, providing for the




(*a*)

termination of 20 pairs in each half. The left-hand half (from the face) is used to terminate the exchange or "E-side" cable and the right-hand half the distribution or "D-side" cable. Sketch (*b*) is a section through one "cell" of the terminal block showing the arrangement of the parts. For bridged or pinned connexions the mirroring of the two halves permits the copper pin to pass right through the block, thus affording internal connexion of adjacent conductors of the E-side and D-side cables. When the appropriate pressure screws are tightened in the threaded metal insert, the end of the screw bears down onto the plastic pressure




(*b*) (*c*)

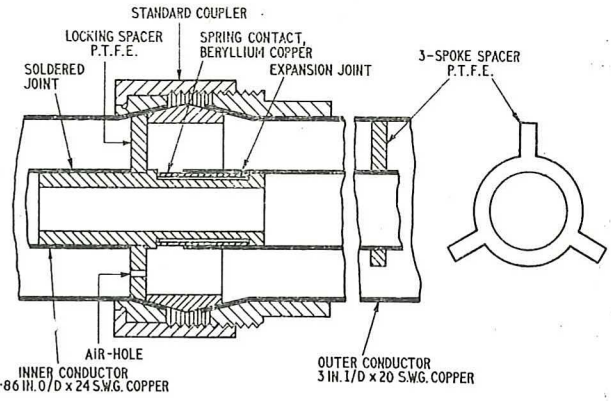
block, which in turn forces the copper pin into intimate contact with the serrated edge of a terminal tag or contact member. Thus electrical connexion is established between the adjacent pairs. It may be seen from sketch (*c*) that the pressure block is designed to operate simultaneously on both conductors of a pair. If it is desired to connect non-adjacent pairs only sufficient of the jumper wire is stripped (about $\frac{1}{2}$ in.) so that, when it is inserted into the terminal block, contact is only made with the pair on that side.

Assemblies of this type are connected into the local-line network to provide a ready means of interconnecting the main-cable pairs from the exchange to the cable pairs serving the various subscribers' distribution points in a local area, i.e. to provide flexibility points in the network. This leads to considerable economies in the provision and rearrangement of underground plant, mainly due to the economies that may be effected in the provision of expensive main cables between the exchange and the flexibility point.

***Q.* 7.** *Describe the construction of a 75-ohm coaxial feeder suitable for serving a short-wave transmitting aerial. Give typical dimensions and state the factors which determine them.*

**A. 7.** The construction of a 75-ohm coaxial cable suitable for connexion between a transmitter and a short-wave aerial system is shown in the sketch.

Both the inner and the outer conductors are of copper tube, the inner conductor being centralized with respect to the outer by means of a three-pronged insulator, of ceramic or P.T.F.E. material. The spacing insulators are located on the inner conductor by a suitable cement and are spaced at random intervals of approximately 5 ft. Expansion and contraction of the central conductor is allowed for at the expansion joint and successive lengths of cable are joined together by means of the standard coupler.




Current and voltage limitations, as well as attenuation, are factors in the design of the cable. If the r.f. power input to the cable is increased, attenuation losses, and thus current, will increase causing an increase of temperature. If the temperature of the central conductor is raised above the safe limit, damage may be caused to the spacing insulator and soldered joints. The increased voltage across the cable may cause corona to be set up, with resultant breakdown of the dielectric. In order to satisfy the requirements, the design must, therefore, take into consideration the physical size of the component parts, as well as the value of characteristic impedance.

The coaxial cable is largely air-spaced so that the characteristic impedance may be defined sufficiently accurately in terms of the geometric parameters and is given by

$$Z = 138 \log_{10} \frac{b}{a},$$

where $a$ is the external diameter of the inner conductor and $b$ is the internal diameter of the outer conductor. For $Z = 75$ ohms, the ratio $b/a$ is equal to 3·49.

***Q.* 8.** *Give an outline description of the manufacture of a section of 0·62-inch polythene-insulated submarine coaxial cable suitable for a shore-end running up a rocky beach.*

**A. 8.** The submarine cable required is the shore-end type, double armoured. It is manufactured as follows:

(*a*) *Inner conductor.* This consists of a solid annealed copper wire of high conductivity, having a diameter of 0·1600 ± 0·0003 in.

(*b*) *Insulation.* The inner conductor is passed through an extruder under carefully-controlled conditions and is covered with polythene to a diameter of 0·620 ± 0·007 in. The insulated inner conductor (known as the core) is carefully examined and then coiled in tanks of water. After a period of not less than eight hours, the core is tested electrically while still immersed in water.

(*c*) *Outer conductor.* The core is passed through a taping machine and is lapped with six annealed copper tapes. The tapes are applied tightly and uniformly with a left-hand (anti-clockwise) lay. They are of high-conductivity copper, each 0·320 ± 0·005 in. wide and 0·0150 ± 0·0005 in. thick.

(*d*) *Binder tape.* The outer conductor is covered with an overlapping copper tape 1·375 in. wide and 0·0030 ± 0·0004 in. thick, applied with a left-hand lay. This is followed with an impregnated cotton tape 1·375 in. wide and 0·015 in. thick, applied with a slight overlap.

(*e*) *Screening tapes.* The taped cable is screened with five soft-iron tapes which are applied tightly in successive layers. The first tape is applied longitudinally and the remainder with alternate right and left-hand lays. Each tape is 0·0060 ± 0·0005 in. thick and of suitable width. The diameter over the screening tapes must not exceed 0·795 in.

The screened cable is taped with an overlapping cotton tape which is coated on the inner side with compound.

(*f*) *Polythene sheath.* The screened and taped cable is passed through an extruder and covered with polythene. The nominal thickness of the sheath is 0·1 in. and the overall diameter must not exceed 1·025 in. The polythene-sheathed cable is immersed in a tank of water for at least 12 hours and is then tested for insulation resistance.

## LINE PLANT PRACTICE B, 1964 (*continued*)

(*g*) *Armouring.* The polythene-sheathed cable is passed through an armouring machine and double-wire armoured as follows:

(*i*) A layer of jute yarn is applied over the polythene to serve as a bedding for the first layer of armour wires.

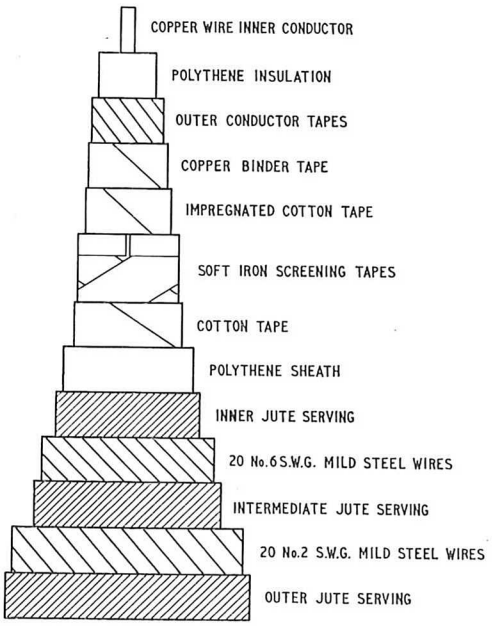
(*ii*) Twenty No. 6 S.W.G. mild-steel wires are applied in an even layer with a left-hand lay.

(*iii*) Two layers of jute yarn are applied with left and right-hand lays respectively to serve as a bedding for the second layer of armour wires.

(*iv*) Twenty No. 2 S.W.G. mild-steel wires are applied in an even layer with a left-hand lay.

(*v*) The armoured cable is served overall with two layers of jute yarn applied with right and left-hand lays, respectively.

The armour wires are dipped in tar or bitumen compound before application to the cable and the intermediate and outer servings of jute are coated under and over each layer with bituminous compound. The make-up of the cable is shown in the sketch.




(*h*) *Testing.* The following tests are carried out on completed cable after 24 hours immersion in water: inner conductor resistance, capacitance, insulation resistance, characteristic impedance, pulse echo, attenuation, high voltage and phase constant.

Shore-end cable which is jointed to lightweight submarine cable, i.e. cable which has a steel-wire centre and no armouring, has a modified make-up in order that the characteristic impedance of the shore end shall match that of the sea cable.

Shore-end cable is manufactured as follows:

(*a*) *Centre strand.* Nineteen ungalvanized annealed mild-steel wires are laid up in two layers to form a circular strand. Each wire has a nominal diameter of 0·036 in. and the diameter of the complete centre strand is 0·180 $\pm$ 0·002 in. The two layers have left and right-hand lays, respectively.

(*b*) *Inner conductor.* The centre strand is passed through a machine which applies a high-conductivity annealed copper tape longitudinally to the strand. The edges of the tape are tightly closed with a box seam. The copper tape is 0·760 $\pm$ 0·005 in. wide and 0·01000 $\pm$ 0·00025 in. thick and the diameter over the complete inner-conductor assembly (except over the box seam) is 0·200 $\pm$ 0·003 in.

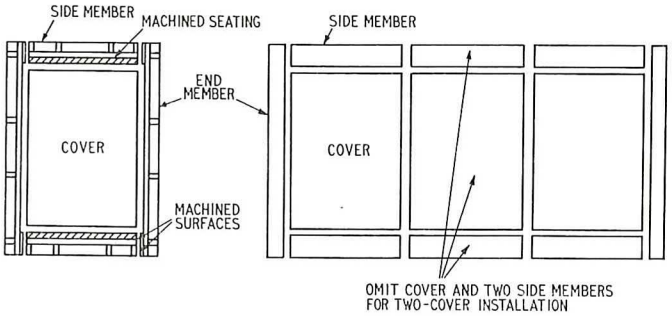
(*c*) *Insulation and subsequent operations.* These are as already described with some small modifications to dimensions.

***Q.* 9.** *Outline the safety precautions that must be observed when working on coaxial cables carrying power at high voltage.*

See A.4, Line Plant Practice B, 1960. *Supplement*, Vol. 54, p. 2, April 1961.

***Q.* 10.** *Describe one type of manhole frame and cover suitable for use in the carriageway, and explain the features of its construction which enable it to meet the loading requirements.*

**A. 10.** The design currently being installed is known as the "Unit-Type Frame and Cover" and may be assembled to give one, two or three cover installations, as indicated in the sketch. The




covers and frame members are made in cast iron and are machined to sufficiently close tolerances to permit full interchangeability.

The side members are of conventional inverted "T"-section design with a machined seating for the cover to rest on and machined ends to butt against mating machined surfaces provided on the "L"-section end members. Bolts passing through the cover are screwed into the side members and clamp them to machined seatings on the underside of the cover. The end members bolt onto the side members and thus form a frame and cover rigidly bolted together. The inverted "T" and "L"-sections provide a substantial flange on the outside of the frame and the side members are equipped with levelling bolts to adjust the whole unit to carriageway level during installation.

The cover, which is a one-piece casting, consists of five beams spanning the frame seatings, with a top plate for supporting the filling material. Above the plate, 1 in.-high ribs are provided, forming a grid of 4½ in. squares which are filled with asphalt. Two thin transverse webs are provided to support the rounded ribs on which the cover slides when being removed from and replaced in the frame.

The main constructional feature of the cover is the five beams, which are shaped to take advantage of the reduction in bending moment at the supporting edges. The beams are also deep to provide a stiff cover, without unduly increasing its weight, and thus to keep the deflexion to a minimum. The position of the neutral axis of these beams is such that the cover is strong enough to withstand the upward deflexion experienced when loads are rapidly applied and removed by fast-moving traffic. The bolting together of frame and cover on fully-machined seatings, prior to installation, should ensure that no rocking of the cover occurs under traffic loadings. Manufacturing tolerances permit small sliding movements of the cover in the frame and on roads carrying fast, heavy traffic it is advisable for the covers to be kept bolted to the frame. The frame members are of ample proportions to resist the rapidly-applied loads of fast, heavy traffic, and to spread the loads onto the concrete base beneath the frame.

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## TELECOMMUNICATION PRINCIPLES C, 1964

**Students were expected to attempt not more than any six questions.**

***Q.* 1.** *When is a transformer said to be ideal? Explain why the magnetizing current of a transformer increases as the frequency of a constant applied voltage is reduced.*

*An ideal transformer has a turns-ratio of 1:2. What is the impedance measured at the primary when a resistance of 2,512 ohms is connected to the secondary? How is this value modified at a frequency of 200 c/s if the inductance of the primary winding is 0·5 H?*

**A. 1.** A transformer is said to be ideal when it has no losses, i.e. when

(*a*) the output power is equal to the input power,

(*b*) there is no leakage flux between the windings, and

(*c*) the impedances of the windings are very much greater than the impedances into which they operate.

To satisfy these conditions, the hysteresis and eddy-current losses in the core must be negligible and the winding resistances must be very small; also the mutual inductance, $M$, must be the geometric mean of the winding inductances $L_1$ and $L_2$. Thus, $M = \sqrt{L_1 L_2}$.

When the secondary winding of a transformer is open-circuited, the impedance measured across the primary, $Z_p$, is that of an inductance,

$$\text{i.e. } Z_p = \frac{V_0}{I_0} = R_p + j\omega L_p.$$

$$\therefore I_0 = \frac{V_0}{R_p + j\omega L_p} = V_0 \frac{R_p - j\omega L_p}{R_P^2 + \omega^2 L_P^2},$$

$$= \frac{R_p}{R_p^2 + \omega^2 L_p^2} V_0 - j \frac{\omega L_p}{R_p^2 + \omega^2 L_p^2} V_0.$$

Thus, the no-load current has two components,

$$I_W = \frac{R_p}{R_p^2 + \omega^2 L_p^2} V_0,$$

which is in phase with the applied voltage, and

$$I_M = \frac{\omega L_p}{R_p^2 + \omega^2 L_p^2} V_0,$$

which lags 90° behind the applied voltage and is called the magnetizing current.

In a good transformer $\omega L_p >> R_p$.

$$\therefore I_M = \frac{V_0}{\frac{R_p^2}{\omega L_p} + \omega L_p} \simeq \frac{V_0}{\omega L_p}.$$

Thus, the magnetizing current increases as the frequency of a constant applied voltage is reduced.

For a loaded ideal transformer, the magnetizing ampere-turns of the primary are equal to the demagnetizing ampere-turns of the secondary,

$$\text{i.e. } N_1 I_1 = N_2 I_2,$$

and the ratio of the primary and secondary voltages is proportional to their turns ratio

$$\text{i.e. } \frac{V_1}{V_2} = \frac{N_1}{N_2} \text{ or } N_2 V_1 = N_1 V_2.$$

$$\text{Then } \frac{N_2 V_1}{N_1 I_1} = \frac{N_1 V_2}{N_2 I_2}$$

$$\text{or } \frac{\frac{V_1}{I_1}}{\frac{V_2}{I_2}} = \frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2}\right)^2.$$

$$\therefore Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2 = \left(\frac{1}{2}\right)^2 2{,}512 = \underline{628 \text{ ohms.}}$$

If the inductance of the primary winding is 0·5 H at 200 c/s, $Z$, the impedance measured across the primary, will be given by

$$\frac{1}{Z} = \frac{1}{628} + \frac{1}{j \times 0{\cdot}5 \times 200 \times 2\pi} = \frac{1}{628} - \frac{j}{628} = \frac{\sqrt{2}}{628} \angle 45°$$

$$\text{Hence } Z = \underline{\frac{628}{\sqrt{2}} \angle 45° \text{ ohms.}}$$

**Q. 2.** *What ranges of frequency are required for the transmission of:*

*(a) speech on a telephone circuit, and*

*(b) music on a program circuit?*

*Why do these requirements differ?*

*A signal in the frequency range 100 c/s to 5 kc/s amplitude-modulates the carrier of a radio transmitter which radiates at 4 Mc/s. A superheterodyne receiver with an intermediate frequency of 500 kc/s receives this transmission. Draw diagrams to show the frequency bands in use: (a) before modulation, (b) on the transmission, and (c) just prior to demodulation. Briefly indicate why each frequency translation is necessary.*

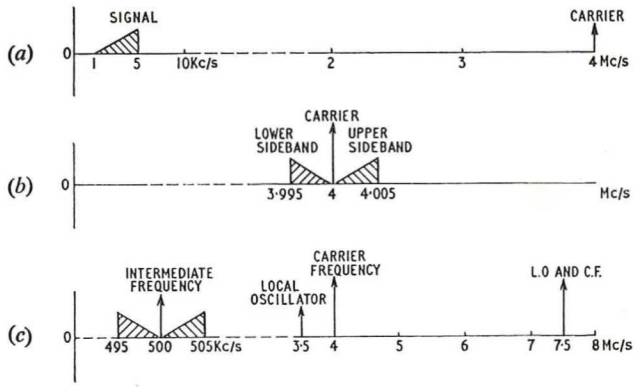
**A. 2.** (*a*) The range of frequencies required on a telephone circuit for the transmission of speech is 300 c/s–3,400 c/s.

(*b*) For music on a programme circuit the range is 50 c/s–8,500 c/s, or 15,000 c/s for special quality circuits.

These are the ranges aimed at in commercial transmissions with the object of using the available frequency spectrum as efficiently as possible.

The range of frequencies which would be ideal for speech should allow all the frequencies produced by the human voice to be transmitted. These normally occur in the range 100 c/s–10,000 c/s. Microphones and receivers capable of operating over this comparatively wide range would be expensive and insensitive. Fortunately, the human ear is very adaptable, thus enabling the frequency band to be considerably reduced and permitting the use of cheaper instruments which rely on mechanical and acoustical resonances for their sensitivity. There is no point in providing a channel which could transmit frequencies above or below those to which the commercial microphone or receiver will respond.

The deciding factor in the reception of music is the human ear. This has an average frequency range of 30 c/s–18,000 c/s, which embraces most of the frequencies generated by musical instruments. In the transmission of music the top frequency can be reduced without noticeable impairment, bearing in mind the performance of commercial receivers and loudspeakers at the upper end of the audio-frequency range.




The diagrams are given in sketches (*a*), (*b*) and (*c*).

Translation from the range 100 c/s–5 kc/s to the 4-Mc/s range is necessary to ensure electromagnetic radiation over considerable distances.

The incoming modulated carrier is mixed with the local-oscillator frequency of the super heterodyne receiver to produce an intermediate frequency (500 kc/s) which can be amplified before detection. This double-modulation process increases the selectivity of the radio receiver and avoids the use of elaborate filters.

**Q. 3.** *A carrier wave E sin ωt is amplitude-modulated to a depth of about 50 per cent by a signal of:*

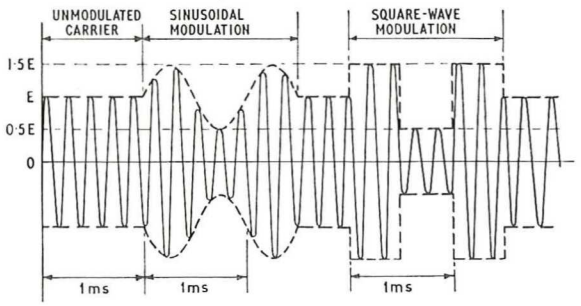
*(a) sine-waveform at 1 kc/s, and*

*(b) square-waveform with a period of 1 ms.*

*Carefully sketch the modulated waveforms, showing for each waveform the periods of the carrier and the signal.*

*Explain briefly what is meant by "side frequencies" and "sidebands", and illustrate these meanings by reference to the two modulated carriers.*

**A. 3.** The waveforms are shown in the sketch.




When a sinusoidal carrier-frequency is amplitude-modulated by a sinusoidal signal at 1 kc/s, the modulated wave contains (at least) three frequencies, namely,

(*a*) the carrier frequency, $\left(\frac{\omega}{2\pi}\right)$ c/s,

(*b*) the lower side-frequency, $\left(\frac{\omega}{2\pi} - 10^3\right)$ c/s,

(*c*) the upper side-frequency, $\left(\frac{\omega}{2\pi} + 10^3\right)$ c/s.

If the modulating wave is $e \sin \phi t$, then the modulated wave can be represented as

$$(E + e \sin \phi t) \sin \omega t = E\left(1 + \frac{e}{E}\sin \phi t\right) \sin \omega t.$$

In the question $\frac{e}{E}$ is 0·5.

$$\therefore E \sin \omega t + e \sin \phi t \sin \omega t =$$

$$E \sin \omega t \; + \; \frac{e}{2}\cos(\omega - \phi)\, t \; - \; \frac{e}{2}\cos(\omega + \phi)\, t.$$

carrier frequency — lower-side-frequency — upper side-frequency

A square waveform can be represented by a series of sinusoidal components which are odd harmonics of the fundamental, thus:

$$y = \frac{A}{\pi}\left(\sin \phi t + \frac{1}{3}\sin 3\phi t + \frac{1}{5}\sin 5\phi t + \ldots\right).$$

When this wave modulates a sinusoidal carrier, it will be realised that the modulated wave contains the angular frequencies $(\omega \pm \phi)$, $(\omega \pm 3\phi)$, $(\omega \pm 5\phi)$ etc., so that a band of frequencies will be produced on either side of the carrier. These bands are known as sidebands. Usually the term sideband refers to the band of frequencies produced when the sinusoidal signal varies over a frequency range, as in speech.

***Q. 4.*** *A coil of inductance 0·1 H and Q-factor 10 is connected in series with a capacitance of 0·1 μF. Calculate:*

(*a*) *the frequency of resonance,*

(*b*) *the resistance of the circuit at resonance,*

(*c*) *the Q-factor of the circuit at resonance,*

(*d*) *the impedance of the circuit at a frequency of 5 per cent lower than the resonant frequency, assuming the resistance to be constant, and*

(*e*) *the resistance of the circuit at resonance if the capacitor had a Q-factor of 100.*

**A. 4.** The impedance of the circuit at an angular frequency $\omega$ radian/s is given by

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right).$$

(*a*) At resonance the circuit is non-reactive, i.e. $Z = R$, and

$$\omega_0 L = \frac{1}{\omega_0 C} \text{ or } \omega_0 = \frac{1}{\sqrt{LC}}.$$

$$\therefore \omega_0 = \frac{1}{\sqrt{0{\cdot}1 \times 0{\cdot}1 \times 10^{-6}}} = \underline{10^4 \text{ radian/s.}}$$

(*b*) The Q-factor of the coil, $Q_L$, is given by the expression $Q_L = \frac{\omega_0 L}{R_1}$.

$$\therefore R_1 = \frac{\omega_0 L}{Q_L} = \frac{10^4 \times 0{\cdot}1}{10} = \underline{100 \text{ ohms.}}$$

(*c*) The Q-factor of the circuit at resonance, $Q_0$, is given by

$$\frac{1}{Q_0} = \frac{1}{Q_L} + \frac{1}{Q_C}.$$

For a pure capacitance $Q_C = \infty$. Hence, for the given circuit,

$$Q_0 = Q_L = \underline{10.}$$

(*d*) At a frequency 5 per cent lower than the resonant frequency, the reactance of the circuit will be

$$\left[\omega_0(1 - 0{\cdot}05)L - \frac{1}{\omega_0(1 - 0{\cdot}05)C}\right] = 1{,}000\left[1 - 0{\cdot}05 - \frac{1}{1 - 0{\cdot}05}\right],$$

$$= -100 \text{ ohms.}$$

So that the impedance of the circuit at this frequency is

$$100 - j100 = \underline{141\angle 45°.}$$

(*e*) If the capacitor had a Q-factor at resonance of 100

$$\text{then } \frac{1}{\omega_0 C R_2} = 100$$

$$\therefore R_2 = \frac{1{,}000}{100} = 10 \text{ ohms.}$$

Therefore the resistance of the circuit at resonance under these conditions will be $R = R_1 + R_2 = 100 + 10 = \underline{110 \text{ ohms.}}$

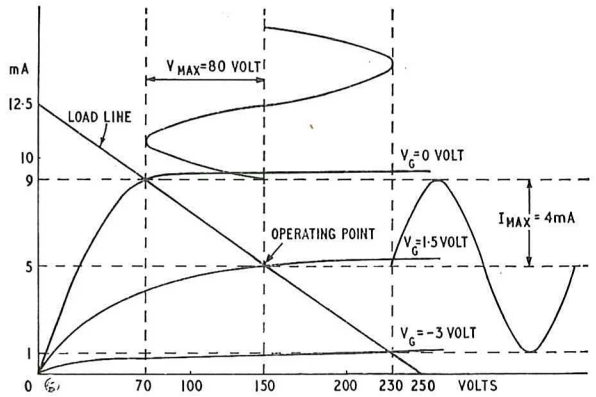
***Q. 5.*** *A pentode valve, biased at − 1·5 volt, takes an anode current of 5 mA through an anode load resistor of 20,000 ohms from a high-tension supply of 250 volt. Sketch a typical family of anode-current/anode-voltage characteristics and insert the load-line and the operating point. Calculate the anode voltage, the anode dissipation and the power dissipated in the anode load.*

*A signal applied to the grid varies the anode current sinusoidally so that the minimum instantaneous value is 1 mA and the maximum instantaneous value is 9 mA. Calculate for these conditions the anode dissipation and the power dissipated in the anode load.*

**A. 5.** With no signal the voltage across the anode-load resistor, since $I_A = 5$ mA, is $5 \times 10^{-3} \times 20{,}000 = 100$ volts, and $V_A = 250 - 100 = 150$ volts. This is the operating point.

When $I_A = 0$, $V_A = 250$ volts, since there is no voltage drop across the anode-load resistor.

When $V_A = 0$, all the battery voltage is effectively across the anode-load resistor, and $I_A = 250/20{,}000 = 12{\cdot}5$ mA. The load-line is drawn through these points, and the operating point, on the typical family of anode-current/anode-voltage characteristics shown in the sketch.




With no signal the anode dissipation is

$$I_A \times V_A = 5 \times 10^{-3} \times 150 = \underline{0{\cdot}75 \text{ watts,}}$$

and the power dissipated in the anode-load resistor is

$$5 \times 10^{-3} \times 100 = \underline{0{\cdot}5 \text{ watts.}}$$

When the signal is applied to the grid, the anode current varies between 1 and 9 mA and the anode voltage varies between 230 and 70 volts.

Assuming a sinusoidal signal, an anode-current swing of 8 mA and an anode-voltage swing of 160 volts the alternating power supplied to the anode-load resistor is

$$\frac{1}{2\sqrt{2}} \times 8 \times 10^{-3} \times \frac{1}{2\sqrt{2}} \times 160 = 0{\cdot}16 \text{ watts.}$$

Thus the total power dissipated in the anode load is $0{\cdot}5 + 0{\cdot}16 = \underline{0{\cdot}66 \text{ watts.}}$

The anode dissipation, with signal applied, is $0{\cdot}75 - 0{\cdot}16 = \underline{0{\cdot}59 \text{ watts.}}$

***Q. 6.*** *What restrictions are placed on the use of equivalent circuits for valves and transistors?*

*Draw the constant-voltage equivalent circuit for a valve and state that:*

$$i_a = g_m v_g - \frac{v_a}{r_a}$$

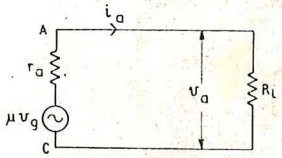
*where $v_g$ is the signal voltage applied between the grid and the cathode and $v_a$ and $i_a$ are the corresponding changes produced in the anode-cathode voltage and the anode current.*

*Draw an equivalent-T circuit for a transistor and show that if the base resistance $r_b$ is negligible:*

$$i_c = \alpha i_e - \frac{v_c}{r_c}$$

*where $i_e$ is the signal current in the emitter lead and $v_c$ and $i_c$ are the corresponding changes produced in the collector-base voltage and the collector current.*

**A. 6.** The main restriction placed on the use of equivalent circuits for valves and transistors is that they must be operating on the linear portions of their characteristics. This generally implies that the equivalent circuits must be restricted to alternating signals of small amplitude. An equivalent circuit for a valve is shown in sketch (*a*)




(*a*)

which applies to small values of alternating currents, $i_a$, and alternating voltages, $v_g$ and $v_a$, $r_a$ is the anode-slope resistance of the valve at the particular operating point.

For this circuit

$$i_a = \frac{\mu v_g}{r_a + R_L} = \frac{\mu v_g}{r_a} \times \frac{1}{1 + \frac{R_L}{r_a}},$$

$i_a$ will be constant and the valve will be a constant-current generator, i.e. the current supplied to the load will be independent of the value of the load, if $r_a \gg R_L$.

For this circuit

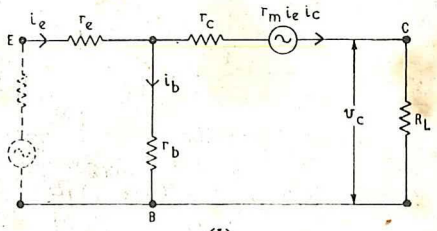
$$\mu v_g - v_a = i_a r_a \text{ or } i_a = \frac{\mu v_g}{r_a} - \frac{v_a}{r_a}$$

$$= g_m v_g - \frac{v_a}{r_a}.$$

*Note:* $\mu = \frac{dv_a}{dv_a}$ and $r_a = \frac{dv_a}{di_a}$.

$$g_m = \frac{di_a}{dv_g} = \frac{di_a}{dv_a} \times \frac{dv_a}{dv_g} = \frac{\mu}{r_a}.$$

The equivalent circuit for a transistor with a grounded base is given in sketch (*b*).




(*b*)

Here $r_b$, $r_c$, $r_e$ and $r_m$ are the slope resistances of the base, collector and emitter and the mutual resistance between the collector and emitter; $i_b$, $i_c$ and $i_e$ are the corresponding alternating currents and $v_c$ the alternating voltage between the collector and base terminals.

For this circuit $r_m i_e - v_c = i_c r_c + r_b i_b$.

If $r_b$ is negligible, $i_c = \frac{r_m}{r_c} i_e - \frac{v_c}{r_c}$,

but $\frac{r_m}{r_c} \simeq \alpha$.

$$\therefore i_c = \alpha i_e - \frac{v_c}{r_c}.$$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